Magic Pinwheel

Grade Levels

This activity is intended for grades 4–6.

Goal

Create a new number system that is created by rotating the origami pinwheel.

Materials

- 8 pieces of origami paper per each student (or pair)
- Blank Paper
- Multiplication tables below

Preliminary Discussion

For most students at this age, they are only beginning to scrape the surface of number systems. Hold a discussion with the students about what kind of numbers they know. Lead them through the natural numbers, integers, and rational numbers. As a "warm-up" activity, ask the students to try and create their own number system with only addition. Be sure to cover these important facts about number systems:

- Additive and Multiplicative identities
- Additive and Multiplicative inverses
- Associativity, Commutativity, and Distribution

Can we create new number systems? Here we will use an origami pinwheel to explore a new number system.

Origami Pinwheel

Please see the instructions to make the pinwheel below. You may also visit:

http://math.serenevy.net/?page=Origami-Figures

Modular Arithmetic

Have the students use their completed model to help answer the following questions:

- Place the pinwheel on a blank sheet of paper and trace. Label the top segment with a zero.
- How many times can you rotate the pinwheel before you get back to your starting point (back to zero)?
- Rotating more than 8 times is not possible on the pinwheel. So what is the equivalent of rotating the pinwheel 9 times?
- If you rotate the pinwheel 17 times, how many times did you rotate it? What about 25? And 57? 108?
- Do you notice a pattern? Try dividing each number by 8, listing the remainder (Don?t use decimals). What do you notice?
- Suppose your pinwheel had 5 sides. If you rotate the pinwheel 9 times, what is that the same as? What about 51, 87, 108, 1024?

This method of counting is called modular arithmetic. We use it everyday when we tell the time. Musicians use it when they play a piece.
Now, instead of using the traditional addition and multiplication, we add and multiply modulo n. This is represented with a three lined equal, $\equiv$, sign instead of '='. We read it as 'congruent to m mod n'. This arithmetic defines a new number system with only a finite amount of elements. Namely $(0, 1, 2, 3, \ldots, n-1)$. We call it $\mathbb{Z}$ mod n and write it as $\mathbb{Z}/n\mathbb{Z}$ or if we're being lazy $\mathbb{Z}_n$. Here the $\mathbb{Z}$ stands for integers. We can do math in $\mathbb{Z}/n\mathbb{Z}$ instead of the real numbers. At this point, have the students learn more about this new number system by completing an addition and multiplication table using the pinwheel. Try it yourself too!

Table for addition mod 8 (left) and regular addition for 0-7 (right)

$+$ mod 8	0	1	2	3	4	5	6	7
0								
1								
2								
3								
4								
5								
6								
7								

$+$	0	1	2	3	4	5	6	7
0								
1								
2								
3								
4								
5								
6								
7								

Table for multiplication mod 8 (left) and regular multiplication for 0-7 (right)

$\times$ mod 8	0	1	2	3	4	5	6	7
0								
1								
2								
3								
4								
5								
6								
7								

$\times$	0	1	2	3	4	5	6	7
0								
1								
2								
3								
4								
5								
6								
7								

Using these tables, have the students address the following questions:

1. Do patterns emerge in the tables built using modular arithmetic?
2. Does associativity hold in $\mathbb{Z}$ mod 8? Commutativity?
3. Is there an additive identity? How about a multiplicative identity?
4. Are there inverses in this system?
5. Does the distributive property hold?
6. Challenge: Are there square roots?

Homework Problems

Practice executing modular arithmetic. Give the students the following problems. Encourage them to come up with a relationship between modular arithmetic and division.

1. 9 mod $8 \equiv$
2. 17 mod $8 \equiv$
3. 51 mod $5 \equiv$
4. $10 + 13$ mod $6 \equiv$
5. $15(3)$ mod $10 \equiv$
6. $52 + 7$ mod $4 \equiv$
7. 29 mod $3 \equiv$
8. $23(7)$ mod $12 \equiv$