Learning Probability

Grade Levels

These activities are intended to be an introduction (or refresher) to probability for 4th - 6th grade students.

Outline and Topics

This entire activity lasts the course of several days (3-5). When broken down, each individual activity applies the basics of statistics (data collection, population, sample, and ratios) and probability (sample space, combinations, permutations) to a context that is more familiar to the students (cookies and Disneyland). Students will learn to reason their hypotheses and apply the mathematical vocabulary and concepts to relevant problems. The activities below are presented to last over the course of 3 days. However, please modify the activity as you see fit for time.

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<th>Brief Description</th>
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<td>Introduction and Discussion</td>
<td>30 mins</td>
<td>Students debate the mathematical claims of ads and commercials use to sell products.</td>
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<td>Chips Ahoy Commercial</td>
<td>5 mins</td>
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<td>Chips Ahoy Sampling</td>
<td>30 mins</td>
<td>Students attempt to verify the claim of the Chips Ahoy commercial through data collection and analysis</td>
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<td><strong>Day 2</strong></td>
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<td>Going to Disneyland</td>
<td>45 mins</td>
<td>Students are introduced informally to the concepts of permutations and combinations through locations of the “happiest place on Earth”</td>
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<td>Permutations</td>
<td>40 mins</td>
<td>Students practice and hone the concept of permutations and combinations visually</td>
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<td>NCTM Game for Permutations</td>
<td>40 mins</td>
<td>Students practice and hone the concept of permutations through a game created by NCTM</td>
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<td>Number 8 Game and Discussion</td>
<td>60 mins</td>
<td>Students begin to learn the concept of probability through this game by developing notions of a sample space (data collection) and patterns (combinations and permutations)</td>
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<td>Creating Games of Chance</td>
<td>45 mins</td>
<td>Students can show-off what they’ve learned by creating a die game of their own (rigged or not)</td>
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Materials and Resources

- Smartboard or Elmo & Overhead Projector
- Paper
- Day 1
  - Day 1 Presentation
  - Several bags of Chips Ahoy
  - Internet Connection
- Day 2
  - Day 2 Presentation
  - Deck of cards
  - Computer access for students
- Day 3
  - Day 3 Presentation
  - Pairs of die for each group of students
- URLs Needed:

Day 1: Chips Ahoy!

Goal for the day

Through this activity, students should learn how to logically justify/discuss with each other multiple claims and devise a plan to prove or disprove such claims. This is a fundamental part of “doing” mathematics as well as being an educated citizen. Through the Chips Ahoy claim, students will learn how to collect and analyze data (specifically by averaging numbers) to prove or disprove the claim.

Introduction and Discussion

Put on the overhead projector a series of commercials and advertisements to show the students. Above are some examples, but save the Chips Ahoy for last! Have the students work through each of the piece, ensuring they understand the point of the exercise is to show that we use should mathematics to approach the claims of others. Here are some guiding questions:

- How was math used in the commercials or advertisements?
- Do you think it is useful to use math when making claims?
- Have you seen any other claims being made in everyday life?
- What can we say about some of these claims? Can we verify them? How?

Take time to write up all the suggestions that the students provide on the board to keep track of the discussion.
Chips Ahoy Commercial

Show the students the Chips Ahoy commercial. Continuing with the theme of the day, ask the students how math was used in the commercial. What is the claim Chips Ahoy is making? Do you think the claim is true or false? Why? Ultimately, the students should determine what the claim is (1000 chocolate chips in each bag) and how they can prove or disprove the claim (count the chips).

At this point, with the students in groups of at least 4, give each group a container of Chips Ahoy cookies (NOT TO BE EATEN!!). Their task will be to determine, on average, the amount of chocolate chips in each container. If the students are unfamiliar with the concept of an average, this would be the time to teach them how to compute an average. Otherwise, have the students determine their own method of counting the chips. Some will count every single chip and others will average across a few cookies. The more parity in counting methods, the better!

Once every group has counted or computed the number of chocolate chips in their bag of Chips Ahoy, take the class average. What was the result? Did Chips Ahoy lie to the consumer?

Day 2: Permutations

Goal for the day

This day is meant to introduce the students to and/or reinforce the concepts of permutations and combinations. The students will first develop the notion of combinations through the Disneyland activity below. Eventually, a natural question will arise regarding the order of travel, at which point one can make the distinction between combinations (order does not matter) and permutations (order matters). These concepts will be reinforced through the remaining activities of the day.

Going to Disneyland

First ask the students to name the location of three Disney Theme Parks. Now have the students write out all the different ways we can visit two of the named three Disney theme parks. A natural question arises of whether or not the order matters. This is a good opportunity for the students to explore and discuss the difference between the two. What if the order mattered, that is, each time a different route was taken it would be considered a different way? What if order didn’t matter? Would that give more or fewer ways? This is an introduction to the idea of permutations. Give the students time to find the difference between the two. Have them list the combinations when order matters and when it does not. The difference will be clear! Have the students who finish continue to the different ways there are to go to all three Disney theme parks?

Four-Card Permutations

Next, with the students in groups of at least 4, pass out the 4 cards for them to visually arrange. Now have the groups record all the arrangements of the four cards they can. Most students will naturally apply an order, another excellent opportunity to point out the difference between permutations and combinations. Provide plenty of time for the students to arrange and record all results. Then share with the class how many each group found. (Write up all the possibilities if the groups have different answers).
NCTM Permutation Game

Here is the link to the NCTM Permutation Game - Bobbie Bear:

http://illuminations.nctm.org/ActivityDetail.aspx?ID=3

After ample time playing, ask the students if they saw a relation or any patterns. You would want the students to eventually get to a point where they see that the number of pants times the number of shirts equals the total outfits. See what types of interesting relations the students came up with on their own and if nobody gets there, guide the students in the direction so that they can connect the dots.

Day 3: Number 8

Goal for the day

Students will play the Number 8 with a pair of die and will learn justifying their responses to questions mathematically as they did on the first day through data collection and build a sample space/recognize patterns through their understanding of permutations.

Rules of the Number 8 Game

- Have each student fold a paper in thirds
- For each column, write “Under 8”, “Exactly 8”, and “Over 8”
- Prior to dice being rolled, the students place a marker on one section of the paper. The student wins if the sum of the faces of the dice is in the category they chose.

Discussion and Constructing a Sample Space

After the students have familiarized themselves with the game and play for awhile, bring them back for a discussion on anything they’ve notice while playing.

- Which section do you think has higher chance of winning? Why?
- Does it matter how many games we played?
- Do you think this game is fair or not? How do you reason?
- Does the order of the dice matter? For instance 1 & 2, 2 & 1?? Why or why not?

At this point, have them create a sample space of all the possible outcomes given a pair of die. This will be a six by six table with the values being the sum of the die.

Patterns and Conclusion

See if the students had a systematic way of writing out the sample space. The patterns will depend on the student's methodology so make sure to have some sort of projector so the student can show their work along with how they wrote it out.
Creating Games

Now that the students learned about the sample space of dice being rolled and rigged games, have the students come up with a game that they can share with the class using their knowledge. If there is extra time, pick some students’ games and play with the entire class.