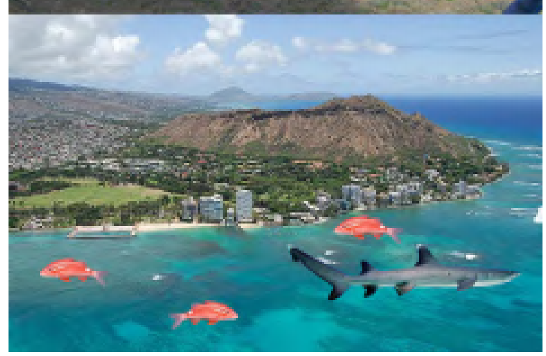
Hawaiian Aquaculture

Grade Levels and Outline

This worksheet is intended for grades $9-12$. This PDF consists of an introduction to the mathematical application, worksheets, and the solutions.

Introduction



The most important fishery management objective is to protect the long term health of the stocks to ensure a good harvest for our grandchildren. Many populations of exploited fish are declining in numbers and size despite the best efforts of fishery managers. Marine protected areas (MPAs) offer a way out of this downward spiral. If some of the larger, more fecund, and genetically more robust fish are fully protected from harvesting, those fish will provide a dependable quantity and quality of offspring. The most effective MPAs protect ecosystem structure and function by including a core of no-take reserves in which extraction of all living organisms is prohibited. Because ocean currents transport eggs and larvae over large distances, networks of no-take reserves or Kapu Zones are needed to achieve the stock rebuilding objective. Reserves are also needed to protect functional ecosystems and habitat areas of particular concern. Restricting fishing in nursery and spawning grounds to rebuild depleted stocks has long been a part of fisheries management in Hawaiʻi. Native Hawaiians were the first to use Kapu Zones as a management told and established caretaker for different areas of land and sea. Hawaiʻi has several marine protected areas on Oʻahu, Hawaiʻi, Lanaʻi, and Maui. Marine Life Conservation Districts (MLCDs) are marine protected areas that may permit some extractive activities, including certain kinds of recreational fishing such as pole-and-line, spear fishing without SCUBA, and certain types of nets. Commercial fishing is generally forbidden. There are MLCDs at Hanauma Bay, Pupukea, and Waikiki on Oʻahu; Lapakahi, Kealakekua Bay, Waialea Bay, and the Old Kona Airport on the Island of Hawaiʻi; Molokini Shoal and Honolua-Mokuleia Bay on Maui; and Manele-Hulopoe on Lanaʻi. Only two, at Hanauma Bay and Waikiki, prohibit all harvesting. No-take marine reserves provide insurance against stock collapse and preserve biodiversity. By prohibiting all harvesting within a designated area, complete protection from both the expected and unexpected effects of extractive activities can be achieved. Since fish in no-take reserves aren't caught, or injured and then discarded, they will survive to grow, reproduce and be caught another day.

Aquaculture Worksheet

Sampling

1. Assume that Hanauma Bay is a closed and protected environment. A fisherman catches, tags, and releases 100 uhu. A week later, he returns and catches 80 uhu, 10 of which are tagged. What is a good estimate for the population of uhu in Hanauma Bay?

2. A month later, he returns and catches 110 uhu, 6 of which are tagged. Now, what is a good estimate for the population size of uhu in Hanauma Bay?

3. A year later, he returns and catches 50 uhu, 4% of which are tagged. Now, what is a good estimate for the population size of uhu in Hanauma Bay?

4. Why must we assume that the bay is closed off from the open ocean? If we do not assume this, what can we say, if anything, about the population of uhu then?

5. Why are permanently protected areas like Hanauma Bay important?

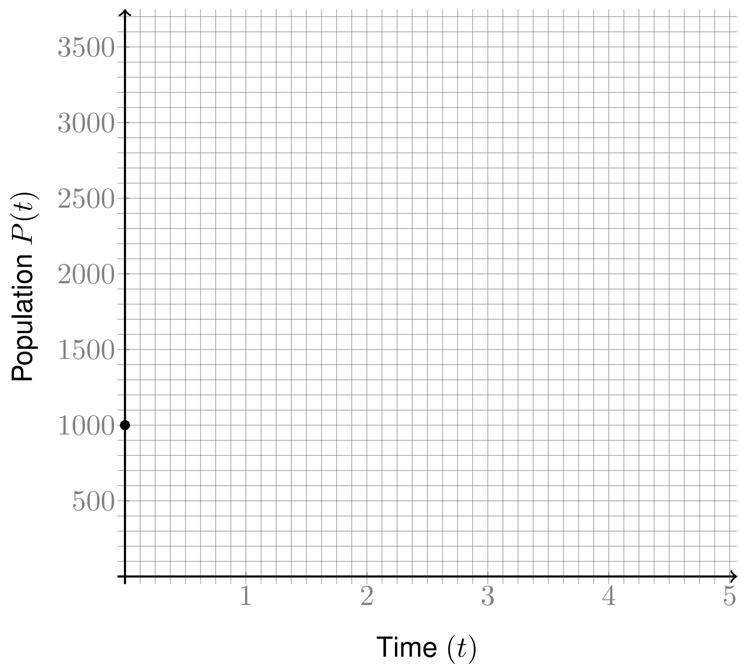
Carrying Capacity and Predator-Prey Relationships

1. The Waikiki-Diamond Head Shoreline Fisheries Management Area extends from the ewa wall of Waikiki War Memorial Natatorium to the Diamond Head Lighthouse, from the high-water mark out to a minimum seaward distance of 500 yards, or to the seaward edge of the fringing reef if one occurs beyond 500 yards. Imagine this is a closed environment. Assume that the population of kumu (or whitest goatfish) is $1,000$ fish, and that food resources for the kumu are unlimited. Kumu are a favorite food of white tip reef sharks. Note that white tip reef sharks reproduce every 12 months, and females give birth to litters of $2-3$ pups, whereas kumu release large numbers of eggs into the water once a month. Describe how the population of both of these fish varies over time.

2. In a closed, protected environment with essentially unlimited resources and no predators, a population of kumu can be modeled by $P(t) = P_0 e^{rt}$, where P_0 is the initial population and r is the rate at which the population grows. Suppose $P_0 = 1,000$ and $r = 0.25$.

   (a) Write the equation that models the population as a function.

(b) Plot the equation as a function of time up to $t = 5$. The initial point has already been plotted.

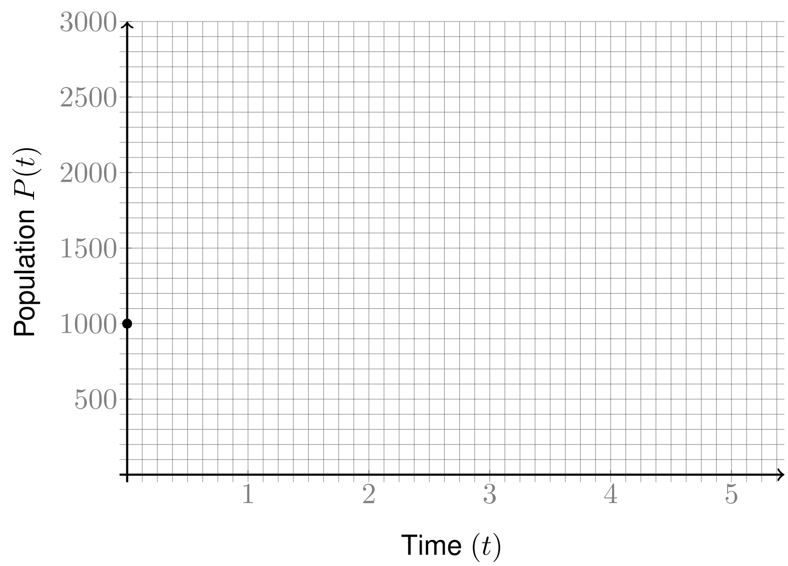


(c) In a protected area like Hanauma Bay, why don't we see exponential growth of the population?

3. In a closed environment with adequate but not unlimited resources and no predators, a population of kumu can be modeled by $P(t) = \frac{K}{1+(\frac{K}{P_0}-1)e^{-rt}}$, where P_0 is the initial population, K is the carrying capacity, and r is the rate at which the population grows. Suppose that $P_0 = 1,000$, $K = 10,000$, and $r = 0.25$.

   (a) Write the equation that models the population as a function of time.

(b) Plot the equation as a function of time up to $t = 5$. The initial point has already been plotted.



(c) Describe the long-term trend of the population. Graphing the function for longer time intervals will be helpful. A graphing calculator will be useful (but not necessary).

(d) Why do we call K the "carrying capacity"? What do we mean by this term?

4. Obviously, kumu have more than one predator. Not only are they favored by white tip reef sharks, but they are also favored by humans! Assuming that kumu are removed from the environment at a rate of 10% of the total population per day by sharks, and 20% of the total population per day by humans. Determine the equation of population decline assuming it is exponential and the initial population of kumu is $P_0 = 1,000$.

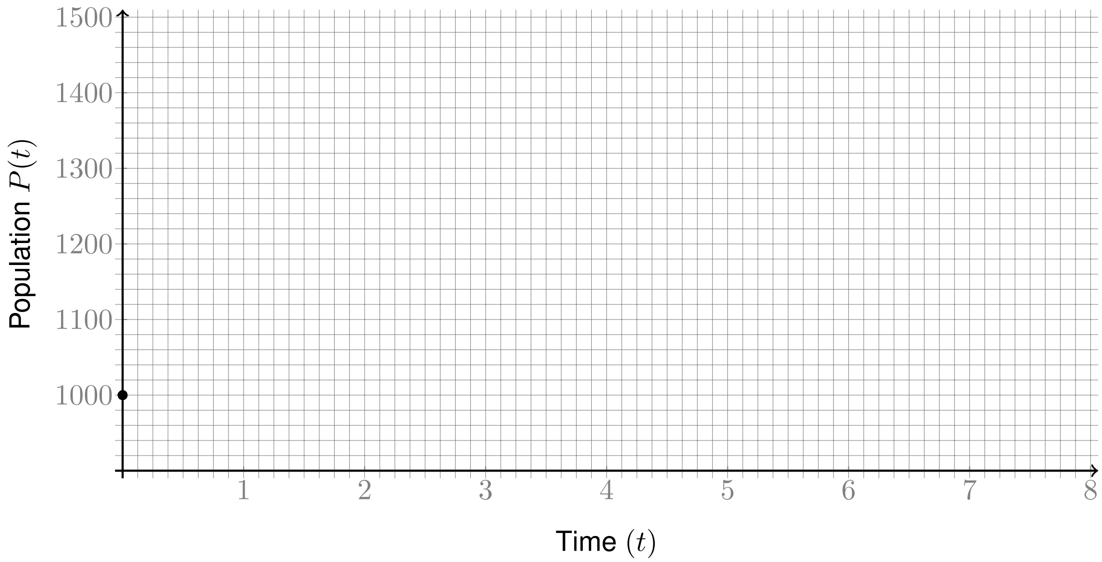
5. The Waikiki-Diamond Head Shoreline Fisheries Management Area is a yearly-alternative protected zone. That is, people can fish from it every odd year, and there is no fishing every even year. Imagine this is a closed environment. Assume that the population of kumu (or whitespot goatfish) is $1,000$ fish. Also, assume that the population growth is modeled by the equation in $\#2$, that sharks are a continuous threat and remove 10% of the total population per day, and that man is a threat on alternative years in which they remove 20% of the total population per day. Assume that population decline of kumu by sharks and man is modeled by the equation in $\#4$.

   (a) Write the equation that describes how the population of kumu changes in the first year.

   (b) Write the equation that describes how the population of kumu changes in the following 4 years. Use the following initial population data for each year to determine the formulas.

n	P_n
0	$1,000$
1	$1,222$
2	819
3	1492
4	670

(c) Plot this function for $t = 5$ years. The initial point has already been plotted.



(d) Describe how the population of kumu varies over time.

(e) What long-term predictions can you make about the population of kumu in the Waikiki-Diamond Head Fishery?

6. Based on what you have learned, what steps should be taken to ensure that the population of kumu are preserved?

Solutions

Sampling

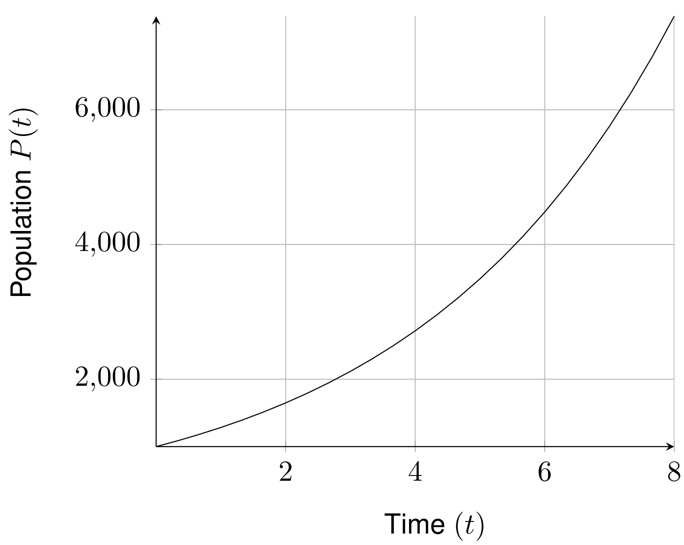
1. There are approximately 70 fish for every 10 tagged. Since there were 100 tagged, we can say there are 700 untagged fish. Adding these two, we get that there are 800 fish as a good approximation. Another way to see this is $(\frac{100}{10}) \cdot 80 = 800$.
2. Using the formula, we get $(\frac{100}{6}) \cdot 110 \approx 1833$.
3. First, 4% of 50 is 2. Using the formula, we get $(\frac{100}{2}) \cdot 50 \approx 2500$.
4. The estimate of the population could be off since the fish tagged could be leaving the environment. We would have to adjust our estimate to take into account the migration of tagged fish.
5. So a given fish population is given a chance to thrive without human intervention.

Carrying Capacity and Predator-Prey Relationships

1. The kumu population, without the sharks, would grow unfettered. With the sharks, they will eat the kumu as fast as they can. If the population of kumu gets too low, the sharks will start to die off. When the population of sharks gets low enough, the kumu will begin to recover, which will cause the shark population to recover. Then, the cycle will restart.

2a $P(t) = 1,000e^{0.25t}$

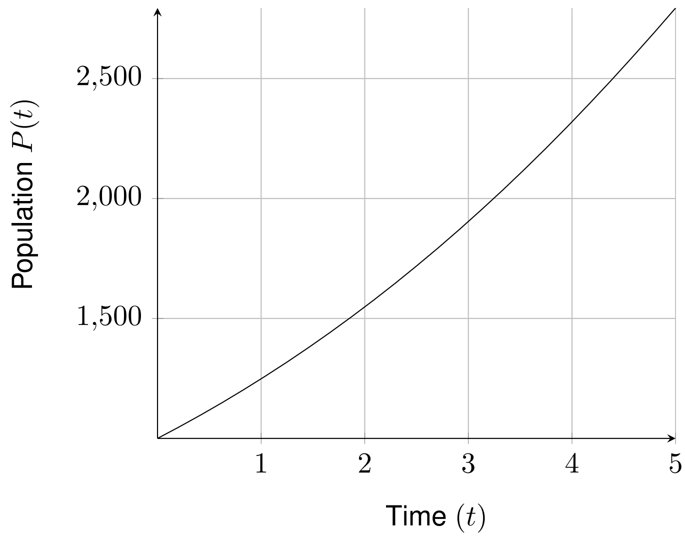
2b Graph:



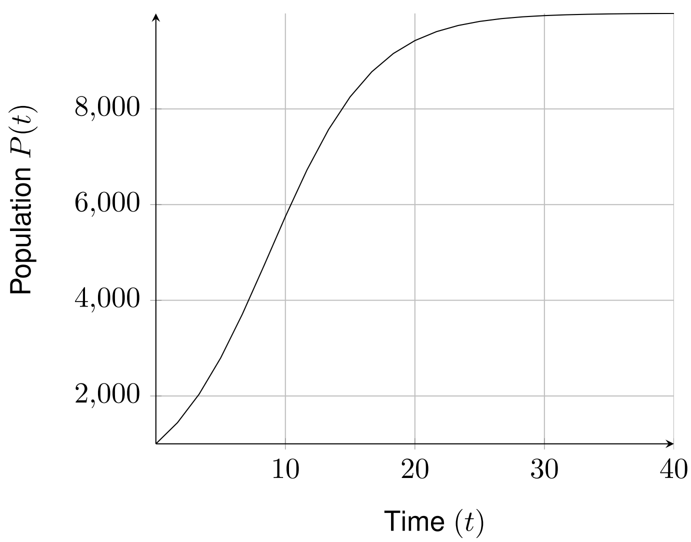
2c Hanauma Bay is not a closed environment, so the fish will migrate away.

3a $P(t) = \frac{10,000}{1+(\frac{10,000}{1,000}-1)e^{-0.25t}} = \frac{10,000}{1+9e^{-0.25t}}$

3b Graph:



3c The population will grow until it reaches a maximum size of $10,000$ fish.



3d We call K the "carrying capacity" because it represents the capacity of fish that the environment will support (or carry).

4 Assuming exponential decay of the kumu population by sharks and humans, we can find the total change in population. The rate that kumu are removed from the environment is $r = (-0.20 + -0.10) = -0.30$. The combined effect is then: $P(t) = 1,000e^{-0.30t}$.

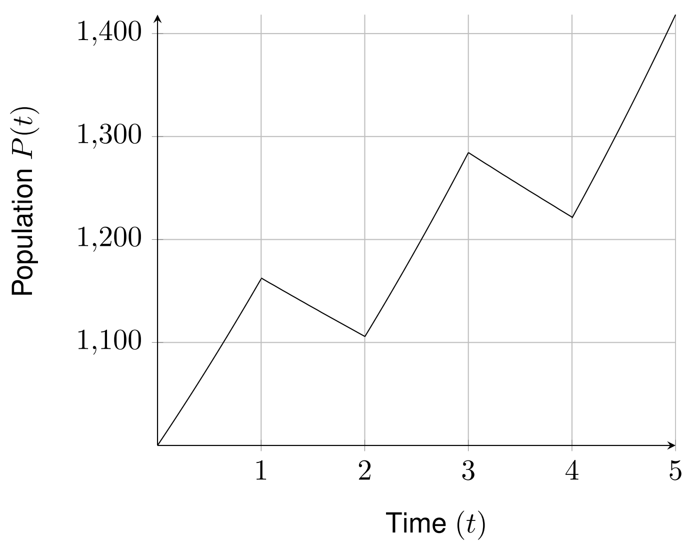
5a In even years, humans are not a threat. Thus, a population of kumu grows at a rate of 25% and is decreased by sharks at a rate of 10%. Thus, the total rate of change in population is the sum of these two rates $r = 0.25 + (-0.10) = 0.15)$. Plugging this into the exponential equation yields: $P(t) = P_0 e^{0.15t}$.

5b In the second year, humans and sharks are threats.

- Thus, using the predator rate of $r = -0.30$ from $\#4$ and the rate of growth for the kumu (which is $r = 0.25$), the combined rate we will use is $r = -0.05$, hence we obtain: $P(t) = P_1 e^{-0.05t} = 1,222e^{-0.05t}$.
- For odd years, the population will be described by the equation: $P(t) = P_n e^{0.15t}$.
- For even years, the population will be described by the equation $P(t) = P_n e^{-0.05t}$.
- The term P_n can be found in the above table.

5c We now know all the equations that describe how the population of kumu changes for the first 5 years. The function then is:

$$P(t)\begin{cases} 1,000e^{0.15t} & 0 \le t \le 1 \\ 1,222e^{-0.05t} & 1 \le t \le 2 \\ 819e^{0.15t} & 2 \le t \le 3 \\ 1,492e^{-0.05t} & 3 \le t \le 4 \\ 670e^{0.15t} & 4 \le t \le 5 \end{cases}$$



5d The population increases on odd years, and decreases on even years.

5e On a long-term trend, it appears to increase with time.

6 Periodically or permanently restricting removal of kumu from their environment by humans will allow fish stocks to increase so that they will always be available and all may enjoy them.