The School and University Partnership for Educational Renewal in Mathematics
An NSF-funded Graduate STEM Fellows in K–12 Education Project
University of Hawai'i at Manoa, Department of Mathematics

Baby Hummer

Grade Levels

This activity can be tailored for 6-12th graders and has been run for each of these grades.

Objectives and Topics

This lesson teaches students about the hummer shuffle, a type of shuffle used in many card tricks. The shuffle has mathematical properties that allow it to "work" for all of the tricks. The students will explore these properties as well as try to prove them. This lesson exposes students to a branch of mathematics that they do not encounter in a typical high school curriculum, and shows them that mathematics exists outside the classroom.

Materials and Resources

1. A deck of cards per student

2. Pen or pencil

3. Paper for each student

Ten Card trick

Use this as a warm up to the baby hummer trick. The students will need some time getting used to hummer shuffling. Give every student a deck of cards and explain the hummer shuffle to each of them, have students practice until they feel confident they are doing it correctly. The following steps are a guide to the ten card trick as well as some discussion questions for the students:

1. Take a deck of 10 cards all face down.

2. Hummer shuffle them as many times as you would like.

3. Lay the cards out with even positioned cards in one row and odd positioned cards in the other.

4. There will the same amount of face up cards in even positions as odd positions.
   Question: Why is this so?
   Let students think about it, it may help to pair them up for this question and work together.
   Answer: If you think about the instructions of the hummer shuffle, when you start all cards are face down and therefore the amount of face up cards at even positions=number of face up cards at odd positions.

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They both are zero. Taking a random cut does not change this. Now turning over the top two cards looks like this

$$\bar{1}, \bar{2}, 3, 4, 5, 6, \ldots, 2n$$

where a bar corrisponds to face up card. Doing another hummer shuffle there are only so many ways to cut. One way could be to just cut a single card off the top of the current deck and then turn over the top two, in which case we will have

$$\bar{3}, 2, 4, 5, \ldots, 2n, \bar{1}$$

where there are still the same amount of face up cards in odd and even positions. The only other way, besides a no card cut, is the both top cards are cut together with any amount of other cards. After the cut and turn over two the deck will look something like

$$\bar{18}, \bar{17}, 19, 20, \ldots, 2n, \bar{1}, \bar{2}, 3, 4 \ldots$$

there will be four face up cards, two even and two odd positioned. The regularity is forced again. If you keep hummer shuffling a similar process will show that the regularity is always forced.

5. Now flip over all the cards in the odd or even row.

6. There will be 5 cards face up and 5 cards face down.
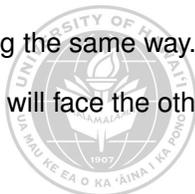   **Question:** Why does this work?
   Have students partner up again to discuss and explore this question.
   Answer: After any number of hummer shuffles there will be some number of face up cards. Let A be the number of face up cards among the 5 cards at even positions. There must be 5-A face down cards among the even positions since there are 5 cards in even posiitons. By what we know the same holds for the cards in odd positions, A face up and 5-A face down. If you remove the cards at odd positions and turn them over this gives 5-A face up cards to add to the A face up cards at teven positions. This makes $(5 - A) + A$=5 face up cards in all. Of course the other 5 cards are face down and the conclusion is reached.

Ten Card Trick 2

Here is an extra trick for the students that "piggy backs" on the first ten card trick. This trick has more of a wow factor to it and the students enjoy it. Have the students pair up and practice the trick to each other.

1. Take a deck of 10 cards that alternate face up face down.

2. Pass the deck to a partner and have them hummer shuffle out of sight (under the table or close your eyes).

3. Take the deck back and hold it behind your back, do not look at the deck.

4. Take out every second card, turn it over, then place it on the table.

5. All of your cards will be facing the same way.

6. The remainder of your cards will face the other way.

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Baby Hummer Trick

Partner the students up again, or let them keep their old partners, and have them learn the baby hummer. With a partner gather a deck of 4 cards, chose one person to be the performer and one person to be the reader. The reader reads the instructions to the performer. After you are done switch roles and do it again.

1. Sneak a look at the bottom card but do not show the reader.

2. Take the top card and place it on the bottom.

3. Turn the current top card face up

4. Cut the deck

5. Turn the top two cards over

6. Cut the deck

7. Turn the top two cards over

8. Cut the deck

9. Turn the top two cards over

10. Cut the deck

11. Turn over the top card and put it on bottom

12. Put the current top card on the bottom without turning it over

13. Turn the top card over and leave it on top

14. Spread the cards out, three cards will be facing one way and your card with be facing the other.

Questions

Have the students discuss the following:

1. What do you think is the "magic" behind this trick?
   Answer The "magic" is in the first and last set of directions. The hummer shuffles simply perserve this order.

2. Can you make this trick work with more than 4 cards?
   Answer If you try with anymore than 4 cards you will quickly see this trick cannot work. There always needs to be an odd card out.

3. Does it matter if the amount of cards is odd or even?
   Answer The properties of the hummer shuffle only hold for an even amount of cards so yes it matters greatly.

4. Does it matter how many times you "cut and turn over two?"
   Answer Because of the properties of the hummer shuffle you can cut and turn over two as many times as you would like and it changes nothing.