Bloodlines

Grade Levels

This activity is intended for grades $9 - 12$.

Introduction

The ancient Hawaiian people kept no written records. Other than the petroglyphs, they knew no written language. Yet they lived with a sophisticated hierarchical system of land divisions, a complex classification in ranks from commoner to highest chief, and a detailed genealogy. To keep track of this vital knowledge, any transition that might be of importance, either to others or to future generations, had to be memorized and passed on. To aid with memorizing, a system of verses emerged which, over the years, developed into an ingenious art form. The verses were known as the "oli", chants. They recorded the history of the land and the lineage of the aristocracy. The chants were crucial for the continuation of the political, social, economic, and ecological system of the Hawaiian world. After all, one's position in Hawai'i depended on one's rank, and ones rank was determined by blood descent. The genealogy was often the only evidence of one's ancestry. It linked a person to all the ancestors, and through this one could show how much sacredness and royal blood had accumulated. Only specially trained kahunas (masters) could haku (compose) and memorize the long chants of aristocratic lineage. When attached to the court of a chief, they often chose others to help them with the careful editing work required to achieve the highest possible power in each chosen word. But in families of lesser rank, it was the firstborn child, the hiapo, who was expected from an early age to memorize all the family's knowledge that had to be preserved.

Problems

1. Given relatives as listed below, determine the genetic percentages in fraction and decimal form of the most recent relative. Assume that all relatives either not mentioned in the current generation or above the mentioned generation are of pure blood.

	Relationship	Decimal	Fraction
a	1 parent hā'oli		
b	Both parents hapa hā'oli		
c	3 grandparents hā'oli		
d	1 grandparent hā'oli		
e	2 grandparents hapa hā'oli		
f	1 grandparent hā'oli, 1 grandparent hapa hā'oli		
g	5 great-grandparents hā'oli		
h	1 great-grandparent hapa hā'oli, 6 hā'oli		
i	3 great-grandparent hapa hā'oli, 1 hā'oli		

Is there a general formula to determine genetic percentage based on the number of ancestors of different backgrounds?

2. Given the following genetic percentages of Hawaiian blood, build a family tree that meets this criteria on a separate piece of paper (see Resources pages). It will be helpful to first compute equivalent fractions in lowest terms.

	Percentage of Hawaiian Blood	Decimal	Fraction
a	50%		
b	25%		
c	75%		
d	12.5%		
e	62.5%		
f	34.375%		

3. Suppose that one of the sailors on the H.M.S. Endeavor, piloted by Captain James Cook, came ashore at Kealakekua Bay in 1779 and fathered a child with a Hawaiian woman. Since that time, the bloodline has been pure. Assuming that there have been 12 generations since that napa hā'oil child was born, what percentage Hawaiian blood would a child in the 13^{th} generation be? *(Use what you have learned.)*

4. Can a person have a percentage equal to $100\frac{A}{B}\%$, where $B \neq 2^n$ and $0 < A \leq 2^n$ (where A and n are both while numbers and $n \geq 0$)? Please explain.

5. In the future, scientists discover a planet that has a species with 3 distinct sexes, call them "male", "female", and "mafemale". Each of the sexes is required to produce an offspring. What are the possible genetic percentages in this scenario for each successive generation? What is the general formula?

Resources

Below are a list of tables, that can be helpful to students when solving Problem 2.

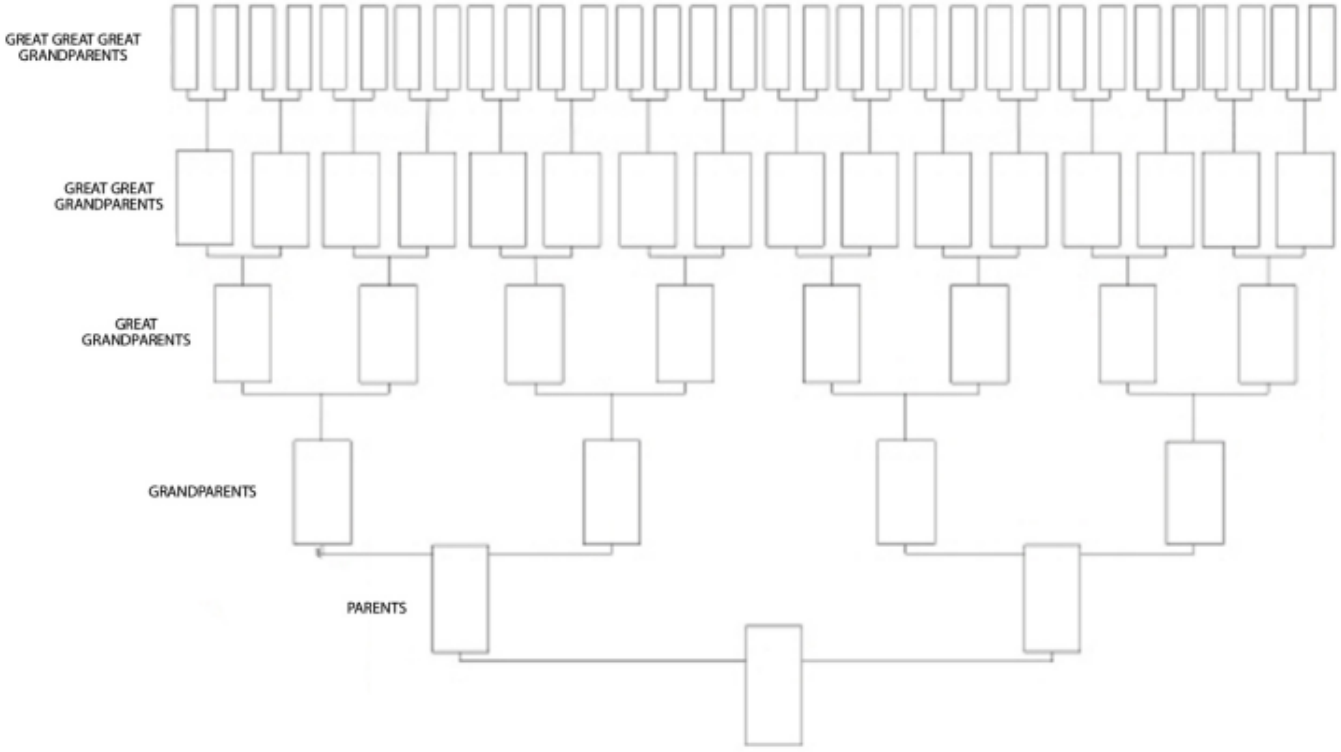

Figure 1: Problem 2a

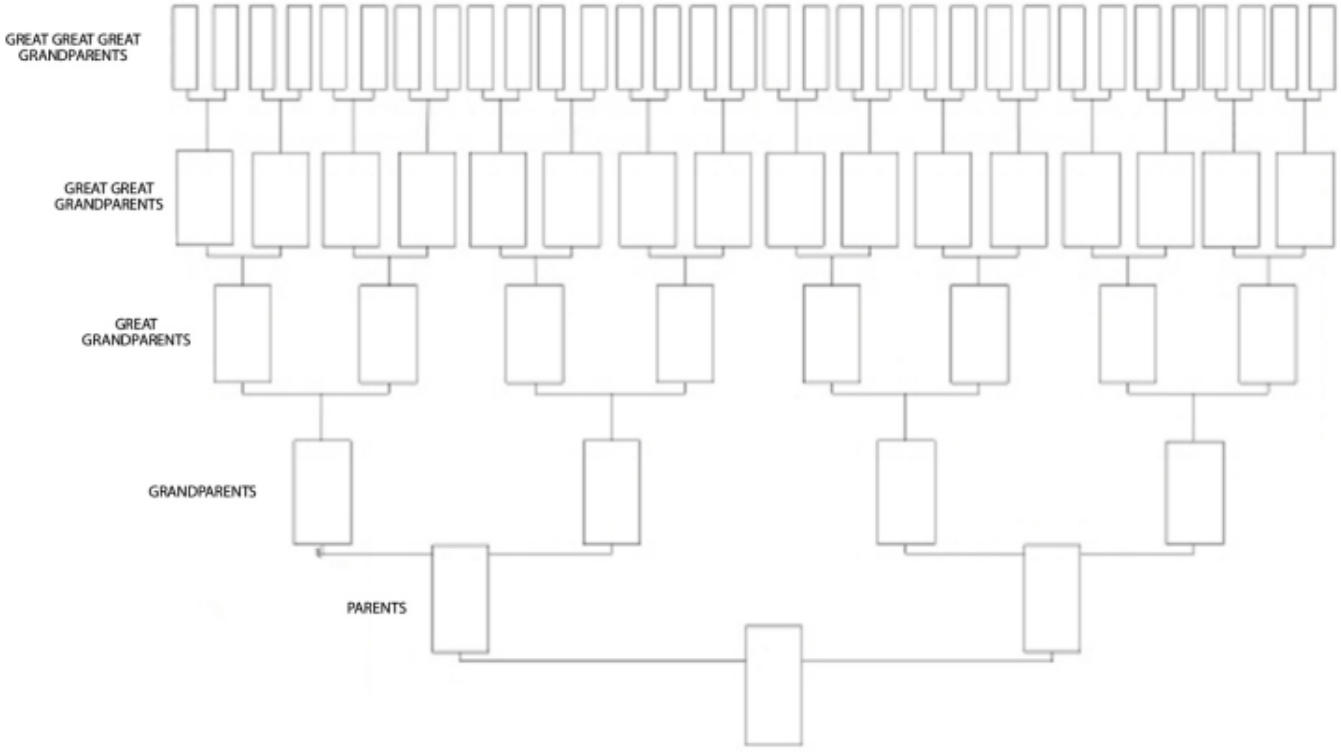

Figure 2: Problem 2b

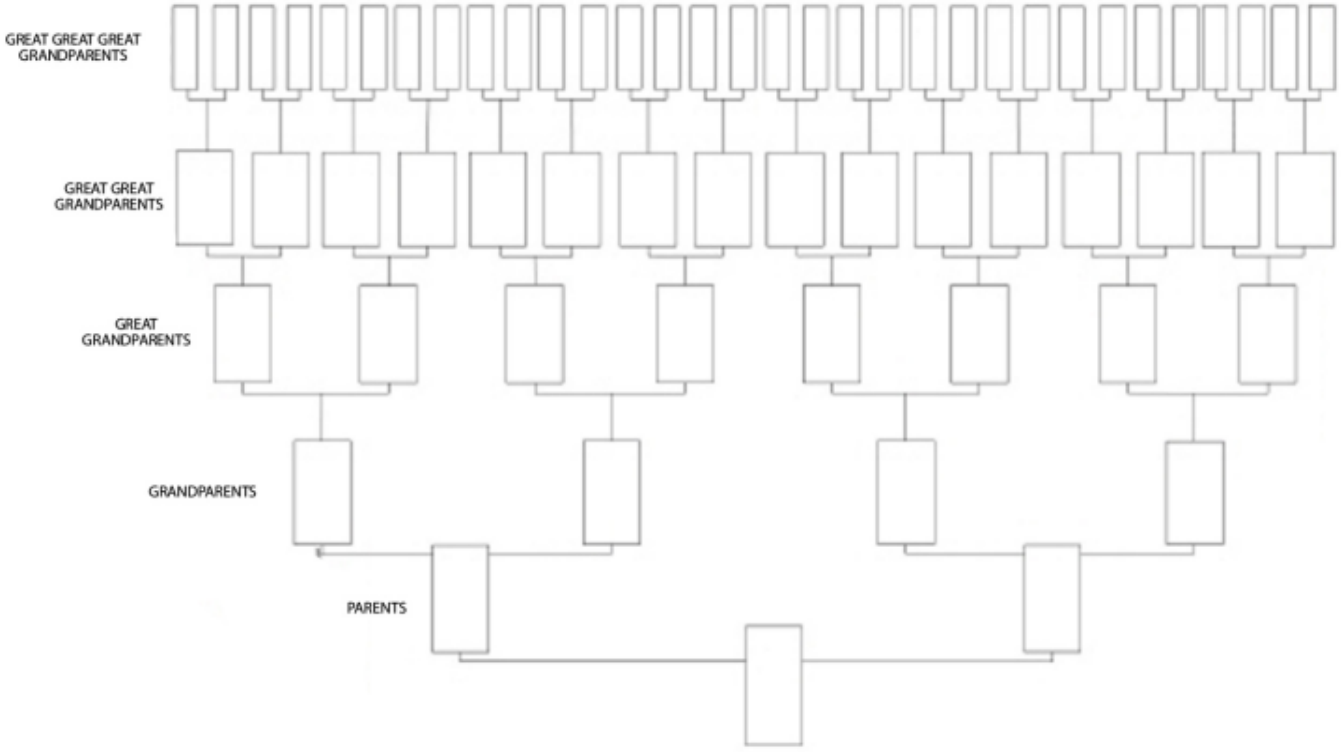

Figure 3: Problem 2c

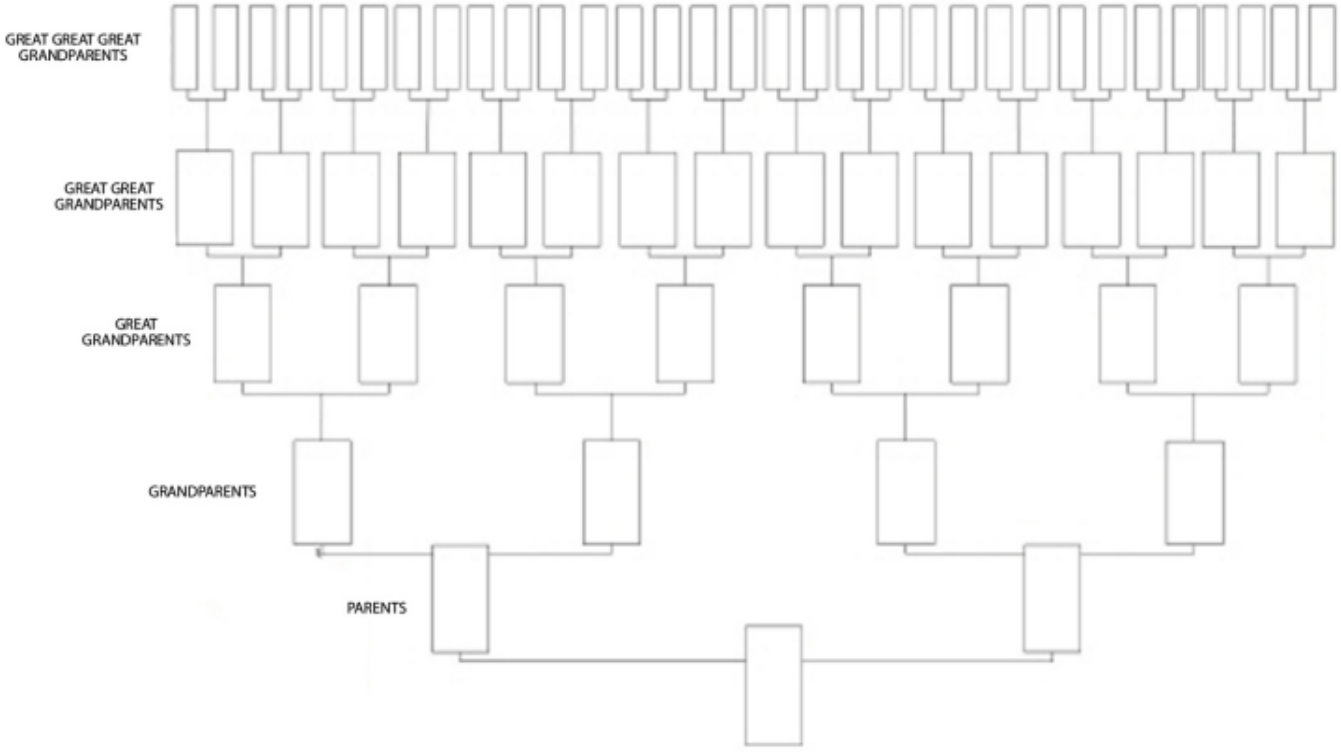

Figure 4: Problem 2d

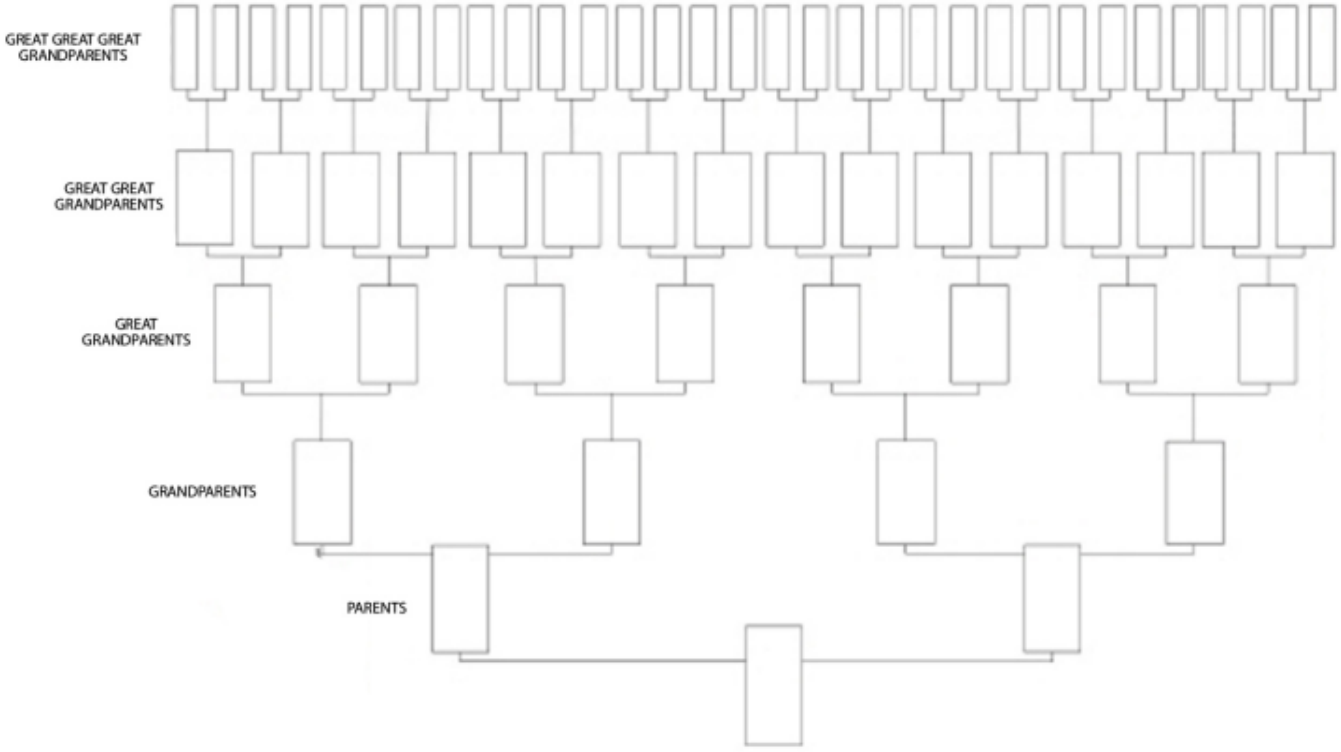

Figure 5: Problem 2e

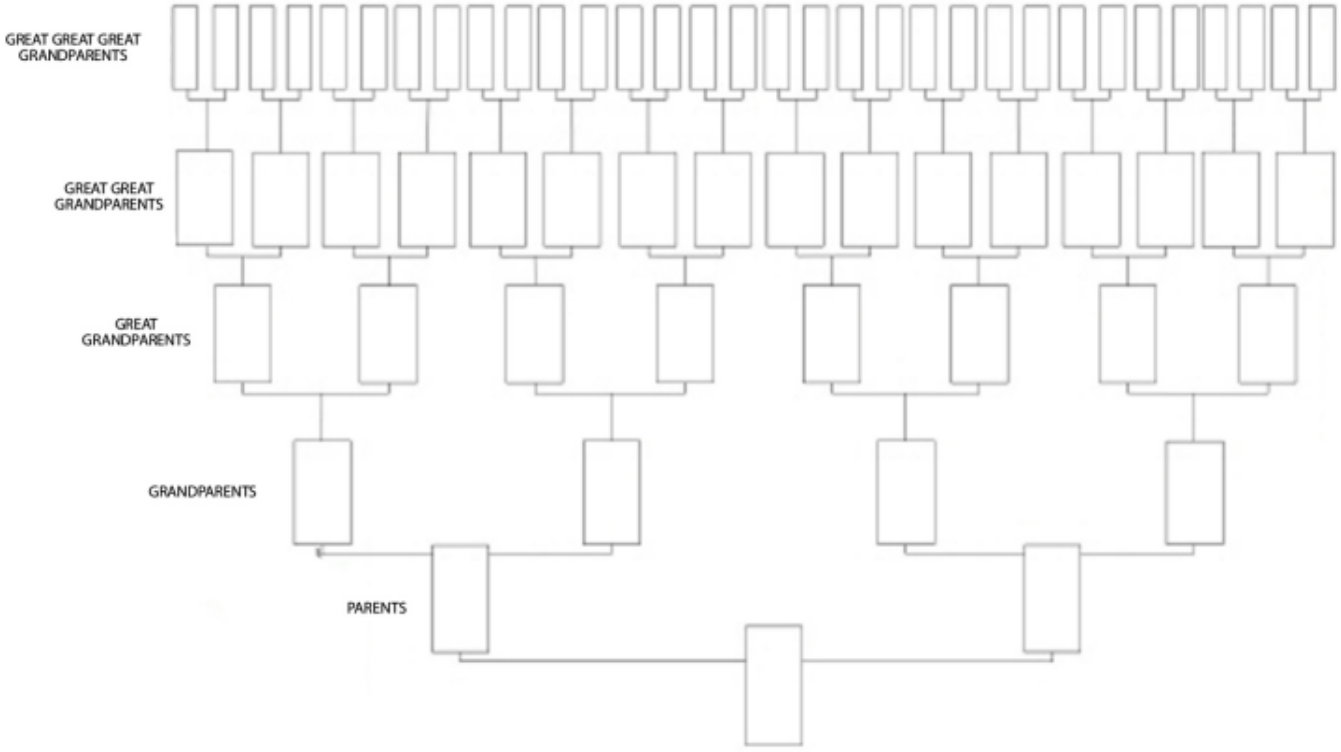

Figure 6: Problem 2f

Solution

1. Table:

	Relationship	Decimal	Fraction
a	1 parent hā'oli	0.5	$\frac{1}{2}$
b	Both parents hapa hā'oli	0.5	$\frac{1}{2}$
c	3 grandparents hā'oli	0.25	$\frac{1}{4}$
d	1 grandparent hā'oli	0.75	$\frac{3}{4}$
e	2 grandparents hapa hā'oli	0.75	$\frac{3}{4}$
f	1 grandparent hā'oli, 1 grandparent hapa hā'oli	0.625	$\frac{5}{8}$
g	5 great-grandparents hā'oli	0.375	$\frac{3}{8}$
h	1 great-grandparent hapa hā'oli, 6 hā'oli	0.1875	$\frac{3}{16}$
i	3 great-grandparent hapa hā'oli, 1 hā'oli	0.6875	$\frac{11}{16}$

If you have all of the percentages of Hawaiian blood of a given generation, add the percentages of each member from that generation and divide by the number of ancestors in that generation.

2. Table:

	Percentage of Hawaiian Blood	Decimal	Fraction
a	50%	0.5	$\frac{1}{2}$
b	25%	0.25	$\frac{1}{4}$
c	75%	0.75	$\frac{3}{4}$
d	12.5%	0.125	$\frac{1}{8}$
e	62.5%	0.625	$\frac{5}{8}$
f	34.375%	0.34375	$\frac{11}{32}$

3. This child would have $2^{12} - 1 = 4095$ ancestors of Hawaiian blood and one napa hā'ole ancestor. Using the formula from Problem $\#1$, the percentage would be:

$$\frac{4095 \cdot (1.00) + 1 \cdot (0.5)}{4096} = 0.999877$$

or 99.877% Hawaiian blood. Equivalently, we could say that this childe has $2^{13} - 1 = 8191$ ancestors of Hawaiian blood and one hā'ole ancestor. By the formula, the percentage would be:

$$\frac{8191 \cdot (1.00) + 1 \cdot (0.00)}{8192} = 0.999877$$

or 99.877% Hawaiian blood.

4. No, this cannot occur. The value of B must be equal to 2^n where n is a whole number greater than 0. This is to signify that a person comes from exactly 2 ancestors, each of a different sex.

5. The possible percentages of any generation are equal to $100\frac{A}{B}\%$, where $B = 3^n$ and $0 < A \leq 3^n$ (where A and n are both whole numbers and $n \geq 0$).