Card Permutations

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February 13, 2013

1 Introduction

The study of permutations has great breadth in mathematics. Understanding permutations is a requirement in applied mathematics, however it is also a familiar step into pure mathematics. For example, in applied math, sorting algorithms are used to optimize algorithms that require sorted lists. The basic function of a sorting algorithm is to output a permutation of the inputed list. Another way to look at permutations is as a group, and thus, studying permutations is a nice introduction to group theory which leads to many areas of pure mathematics. A basic understanding of permutations as a group helps students familiarize the abstraction of group theory.

In this lesson, students will work with playing cards to begin getting comfortable with the idea of permutations. Once comfortable, the students can use their knowledge of finite number systems and modular arithmetic to form a structure for how the permutations interact with each other. This lesson can be used as a springboard for studying basic group theory at the high school level, which is a good way to broaden students' view of mathematics.

2 Length and Objectives

The lesson is designed for a 90 minute session. The objective of this lesson is to have students develop an abstract structure that explains permutations of a fixed number of objects.

3 Prerequisites

Pattern recognition. Understanding the division algorithm thoroughly is helpful.

Note: This lesson was designed as a follow up to an introduction to modular arithmetic. This is because the heart of this activity is repeating patterns, which can be described more sophisticatedly using the theory of modular arithmetic.

4 Grade Levels and Topics

This lesson can be applied to any grade 6-12 math class. The topic of this lesson is modular arithmetic.

5 Common Core Standards

(Note: This lesson has only been aligned with high school standards.)

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Modeling

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods.

A-CED: Creating equations

Crate equations that describe numbers or relationships.

1. Create equations and inequalities in one variable and use them to solve problems.
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

F-IF: Interpreting Functions

Understand the concept of a function and use function notation.

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

Interpret functions that arise in applications in terms of the context.

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

F-BF: Building Functions

Build a function that models a relationship between two quantities.

1. Write a function that describes a relationship between two quantities. (a., b.)

2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

6 Procedure

Time	Procedure
10 minutes	The lesson begins with the presentation of the problem on the first worksheet. Following the presentation, the teacher will lead a problem solving discussion in which students try to isolate key features of the problem, and how they may relate to previous work. (Remember this lesson is an add on to another, although it can be self-contained.)
45 minutes	Students are broken up into groups and given time to consider the two worksheets and find strategies to answer the given questions. The questions are focused on two cases that are more simple than the original problem.
25 minutes	Once the students have worked through the questions. The teacher will lead another discussion, in which, the students share their results. The teacher should note that many strategies will answer the questions but some are more efficient than others. This is also the time when the teacher can relate the lesson to modular arithmetic if the students are well prepared.
10 minutes	The lesson ends with the students working in their groups to solve the original problem, and possibly, creating their own problems.

The timeline of the lesson is highly dependent on the age and skill level of the students participating and should be adjusted accordingly.