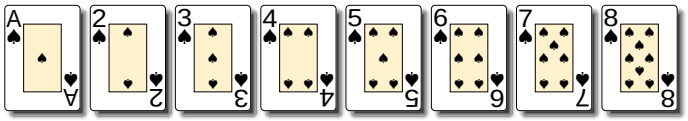
Mathemagician

Name:

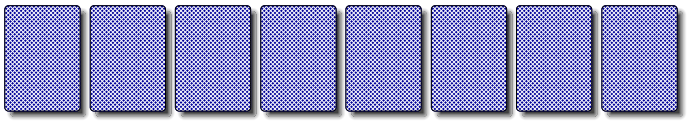
Date:

Problem

Consider the following problem: A magician with extremely fast hands presents a crowd with a challenge. He claims that he can shuffle 8 cards, including one ace, so fast that no person will be able to find the ace when he is done. You become curious and watch for a little while. He always begins by presenting the 8 cards.



He then turns them over



and begins shuffling them as fast as he can. Once finished, he asks a person from the audience to find the ace. The crowd is very unsuccessful.

At first you are blown away by his speed, but using your problem solving skills, you begin to dissect the magician's technique. You notice that there is a pattern in the way the magician shuffles the cards, however, he does not repeat the pattern the same amount of times in each presentation. Since there is a pattern, you know that given enough time you can determine where the ace ends. Unfortunately, you do not have that much time on the spot. Finally, you decide that you have to create some sort of mathematical structure so that you can make an easy calculation to solve any of the magician's shuffles.

After jotting down a few notes, inspiration strikes and you're ready to challenge the magician. You step up to the table, "I would like to try." He shuffles the cards with a smile on his face ready to stump another hopeless victim. When he is done, you make a quick calculation using your well thought out structure and point out the ace. But before he turns it over, you tell him exactly where all the cards have ended. He finds, to his surprise, that you are exactly right and exclaims, "Who are you!?"

You reply, "I'm a mathemagician."

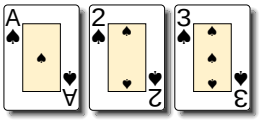
Directions

As mathematicians, we realize that solving this problem with all 8 cards may be too demanding. Therefore, we begin with a simple similar case. First, try to solve the problem with 3 cards. Then use the structure built for the 3 card case to solve cases with more cards. Answer the following questions.

Questions

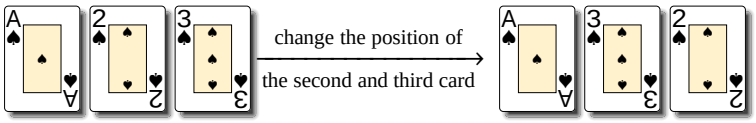
(1) What are all the possible orderings in which you can put 3 cards? How many are there?

Example:



(2) Starting with  explain how you can get to each other ordering (i.e. which cards should move where). These "re-orderings" are called *permutations*.

Example:



(3) Choose one of your permutations and apply it 10 times recording each result.

(4) Will any other permutation provide a similar pattern as the one you chose? If so, which one(s)?

(5) Group the permutations based on the type of pattern they provide.

Think about it! How can your knowledge of modular arithmetic describe what is happening?

Bonus

(A) Classify the permutations (based on "pattern type") of 4, 5, . . . cards.

(B) Solve the magician's challenge if he repeats the following move 100 times