Introduction to Modular Arithmetic: Clock Math

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1 Introduction

High school mathematics often limits itself to studying the real numbers and the usual arithmetic it entails. However, there are various other number systems one can study, some with very different types of operations. In this lesson, a new view is taken on a number system high school students use everyday. The number system we are talking about is the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$. We want to study how this set relates to the way we tell time and how addition is preformed on this finite set. Obviously, the definition of addition must drastically change for a finite set as opposed to an infinite set.

This comfortable setting makes an easy transition into studying modular arithmetic. A place where students deal with finite number systems. This type of study will open students to the idea that number systems and their structure are not absolutes, but rather, the product of well chosen definitions and axioms.

2 Length and Objectives

The lesson is designed for a 90 minute session. This lesson has two main purposes in mind. The first is to introduce students to a new number system and one of its, possibly many, operations. The second is to have students explain this exploration in mathematics to each other, as if they were true mathematics researchers.

3 Prerequisites

Students need a basic understanding of divisibility and the idea of a greatest common divisor.

4 Grade Levels and Topics

This lesson can be applied to any grade 6-12 math class. The topic of this lesson is modular arithmetic.

5 Common Core Standards

(Note: This lesson has only been aligned with high school standards.)

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Modeling

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods.

A-CED: Creating equations

Crate equations that describe numbers or relationships.

1. Create equations and inequalities in one variable and use them to solve problems.
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

F-IF: Interpreting Functions

Understand the concept of a function and use function notation.

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

Interpret functions that arise in applications in terms of the context.

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

F-BF: Building Functions

Build a function that models a relationship between two quantities.

1. Write a function that describes a relationship between two quantities. (a., b.)

2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

6 Procedure

Time	Procedure
10 minutes	The lesson begins with a review of known number systems and how operations are preformed in the various number systems. The teacher should also review general ideas of divisibility and greatest common divisor as a springboard for their exploration.
30 minutes	Students are broken up into groups and given time to consider the worksheet and find strategies to answer the given questions. It is these strategies that the teacher will emphasize as the main focus of the students' presentations.
30 minutes	Each group is given time to present on what problem solving techniques they used to answer the questions. They will also explain what their strategy was and how they would generalize it to any problem of similar type.
10 minutes	The teacher wraps up the presentations with a discussion where students can comment on each others strategies. The goal is to determine which strategies will generalize to any similar problem, as well as among these strategies, which is the most effective.
10 minutes	From here it is an easy transition for the teacher to start talking about modular arithmetic and how it applies to the clock problems. The strategy chosen as the most effective in the discussion should align with the structure defined in the theory. This is, hopefully, where the students feel a sense of satisfaction for their hard work and get a better understanding of how true mathematics works.

The timeline of the lesson is highly dependent on the age and skill level of the students participating and should be adjusted accordingly.