

Dice Trick

Grade Levels

This activity is intended for high school students.

Objectives and Topics

To have the students investigate what is going on, to conjecture, and then finally to work out a detailed mathematical proof. The students might struggle with the formality of a proof, but the earlier they are exposed to mathematical thinking, the better it will be later on. This activity covers concepts in Algebra, specifically the distributive property and F.O.I.L. method.

Activity	Suggested Time	Materials	Preparation
Introduction	15 mins		Have a good understanding of the dice activity to be able to answer any of the questions that the students might have. See below
Activity	10 mins	2 die per student and paper for recording	Have the students double check their calculations. If they struggle, compute product 1 on board as an example.
Discussion	10 mins	Board space to write discussion	See below
Write-up	20 mins	Paper/Journal	See below
Sharing Proofs	15 mins		Have students share their proofs. Keep in mind that there are many ways to prove this. Creativity is welcome here!
Wrap-up	5 mins		Collect material. They have successfully completed a proof!



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Introduction

A proof is a way to show that the answer that you think is correct is actually correct. This is a logical reasoning of why something is the way it is. In our activity, we will come up with guesses and try to see if we can prove it concretely in an elegant manner. So, here are the instructions. Roll a pair of dice; call these A and B. Then calculate the following products.

- Product #1: Product of top numbers on dice
- Product #2: Product of bottom numbers on dice
- Product #3: Product of top of A and bottom of B
- Product #4: Product of bottom of B and top of A

Then finally add the four products. What is the sum for your dice rolls?

Materials and Resources

- 2 die per student
- Blank paper

Discussion

Ask the students what they got as their sum. What do they notice about the entire class? (they all should have the same answer). Ask them what they think about this? Have them come up with a conjecture for our dice trick. Also ask them how they think they can show this? (i.e. prove that their conjecture is correct?) Tell them that this is how mathematicians start up. They do a couple examples to lead them to think that something works and then they go ahead and try to prove that it is correct in all the cases.

Write-up

Have the students write up their proofs. Help them along their thought process if they struggle. Do not lead them toward a certain direction. Try to see how creative they are with coming up with their own way of proving this. A sample proof is provided below. Make sure they include all the details needed in the proof so that the reader can follow without having to ask many questions.

Sample Proof

First of all, we use the fact that the top and bottom of any number on the die equals 7.

Conjecture: The sum of the products equals 49 no matter what the die roll is.

We use letters instead of numbers since we have to prove that for any die-roll combination; we get 49 as

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our sum

Let the top of dice A and dice B have values a, b respectively. So dice $A = a$ and dice $B = b$. Now the bottom of these are $(7 - a)$ and $(7 - b)$. So

- Product #1: ab
- Product #2: $(7 - a)(7 - b) = 49 - 7a - 7b + ab$
- Product #3: $a(7 - b) = 7a - ab$
- Product #4: $(7 - a)b = 7b - ab$

Now the sum $= ab + 49 - 7a - 7b + ab + 7a - ab + 7b - ab = 49$ after canceling out the like terms. Hence our conjecture is true and this is the end of the proof!

UNIVERSITY of HAWAI'I*
MĀNOA



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