# Graph Theory: Lucky or Not?

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## 1 Introduction

Graph theory is a flourishing field in mathematics with numerous applications to everyday problems. Graphs are, in simple terms, a collection of vertices with edges connecting certain vertices. The goal of graph theory is to create a structure on these graphs. This structure usually comes from categorizing graphs based on certain characteristics and then conclude certain attributes for the given categories. For example, a connected graph is a graph where every vertex is connected to every other vertex by some set of edges. This forms a category. A mathematician now wants to understand the shared attributes of all connected graphs. Thus generalizing a solution to what may initially be a specific problem. The applications are clear, since many situations can be modeled by graphs. E.g. maps, floor plans, networks, etc.

## 2 Length and Objectives

The lesson is designed for a 30 minute session. Students will try to solve multiple puzzles while making observations. Then given some basic knowledge of graph theory, they will try to relate it to their puzzles.

## 3 Prerequisites

Students need only have knowledge of counting and an understanding of even and odd numbers.

## 4 Grade Levels and Topics

This lesson can be applied to any grade 2-12 math class. The lesson includes topics in graph theory, mainly Euler's theorem for the existence of Eulerian paths.

# 5 Common Core Standards

(Note: This lesson has only been aligned with high school standards.)

## Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

## Modeling

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods.

### F-IF: Interpreting Functions

#### Understand the concept of a function and use function notation.

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$. The graph of $f$ is the graph of the equation $y = f(x)$.
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

### F-BF: Building Functions

#### Build a function that models a relationship between two quantities.

1. Write a function that describes a relationship between two quantities. (a.)

# 6 Procedure

| Time | Procedure |
|---|---|
| 5 minutes | The lesson begins with the teacher presenting a story. The story outlines a problem in which students, not knowing, need to find an Eulerian path in specific graphs. There will be a short class discussion ensuring the students understand the problem as well as their initial thoughts on whether the problem is solvable. |
| 15 minutes | After the discussion, students will be given time to solve four puzzles that relate to the story. The students should work in pairs. While students are solving the puzzles they should be observing what their solutions have in common. It would be ideal if the students came to some hypothesis without knowing the real mathematics involved. |
| 10 minutes | Then, the class will come together for a discussion. The students will present their hypotheses or at least their observations. The teacher and other students will test the validity of each observation. Finally, the teacher will present the basics of graph theory, in particular, how to find Eulerian paths. Students will have time to answer the question, "How does graph theory relate to our puzzles." |

The timeline of the lesson is highly dependent on the age and skill level of the students participating and should be adjusted accordingly.