Heʻe Hōlua

Introduction

Traditionally, heʻe hōlua (Hawaiian lava sledding) served both as a sport and as a vehicle for native Hawaiians to honor their gods, especially Pele, the goddess of fire. Originally restricted to the aliʻi class of men, the nobility of Ancient Hawaiʻi, it eventually became a sport in which common Hawaiians, men and women, could also participate. After reaching the top of a slope, a Hawaiian would grab the papa hōlua, or sled, (often carved from Kauila or Ohia, hand lashed with coconut fiber, weighing between 40 and 60 pounds and measuring 12 feet long by 6 inches wide), hoist it over his head and sprint toward a run shaped out of a small stream dry bed (or other natural features) that has been filled with pounded dirt, lined with pili grass, then saturated either with kukui nut or coconut oil to make it slippery. They would stand up, lie down, or kneel atop the hardwood sleds and speed down the courses of hardened lava rocks reaching speeds of up to 70 mph on a sled standing only four inches above ground.

Hundreds of years ago, the tree line was much closer to shore and canoes were constructed on site. However, as time passed, trees that were typically used to build canoes became unavailable near the coastlines. It has been conjectured that the original use of a hōlua slide was to transport canoe hulls, hewn into rough shapes in the mountains, to the bay for final finishing. Tremendous amounts of labor were involved in smoothing the path and laying down water-worn stones to aid in transport of the massive canoe hulls. Based on this information, it is no surprise that such smooth paths were utilized for heʻe hōlua. Slides have been discovered at Kahikinui on Maui, in Pāhala on the southeast side of the Big Island, at Kaʻena Point on Oʻahu, and at Keauhou in Kona (called Kaneʻaka) that traverses down the slope of Mount Hualālai to the sea that measures 5,000 feet long, nearly a mile. In 1995, there were only 5 remaining hōlua sites in the world. Now there are upwards of 150 known as archaeologists, anthropologists, engineers and native Hawaiian activists work to revive the sport.

Grade Levels and Topics

This activity is intended for grades 9-12, specifically for students with a sound understanding of algebraic manipulation. For 9-10 grade students, there are extensions of the problems involving trigonometric properties. The Free-Body Construction section as a calculus level activity although it may be modified to adhere to students with minimal to no calculus experience. Both the Google Earth and Slope and Free-Body Construction act as stand alone activities or merged together; here, they are presented as separate activities.

Topics include:

- Modeling with mathematics by using Pythagorean Theorem to determine hōlua track lengths
- Investigate velocities mathematically
- Manipulate the equation for velocity to solve for desired physical quantities
- Use trigonometric properties to determine angles of elevation of the hōlua slides
- Us the Law of Cosines to determine the length of a track comprised of linear segments
- Interpreting results to deduce whether or not they are realistic.
- Modeling with mathematics by discussing/interpreting equations to understand the physical aspects they describe
- Trigonometry, specifically using trigonometric functions to determine quantities
- Given two-dimensional vectors, break them down into their x and y components
- Evaluate, symbolically manipulate, and solve equations
- Take derivatives with respect to one variable of polynomial functions to determine velocities and accelerations
- Anti-derive expressions to obtain velocities and positions

Objectives

Through this activity, students will learn:

1. how different types of mathematical objects/techniques correspond to different physical quantities
2. how to manipulate equations to obtain desired quantities
3. how to reason and justify their ideas
4. how to determine the practicality of a numerical solution.
5. how to apply mathematics to the he'e hōlua

Materials and Resources

Materials

- Pencil and paper
- Access to Google Earth
- He'e hōlua worksheet (see Worksheet Section)

Resources

Google Earth:
http://www.google.com/earth/index.html

Wolfram Mathematica Online Math Engine:
http://www.wolframalpha.com/

Wolfram Mathematica Online Integrator:
http://integrals.wolfram.com/index.jsp

Historical story of the he'e hōlua race between Kaha-wali and Pele, from "Hawaiian Legends of Volcanoes" by W.D. Westervelt:
http://www.sacred-texts.com/pac/hlov/hlov12.htm

Google Earth and Slope

Time	Procedure
5 mins	Historical Introduction & Motivation of the mathematics
45 mins	Google Earth and Slope
15 mins	Discussion for Slope Problems

Historical Introduction and Motivation

Using the introduction above, share with the students the Hawaiian history of the he'e hōlua. Explain to the students how mathematics is used to model actual physical situations, as this is precisely what they will do in this activity. It is important for students to know in addition that mathematics is more than simply manipulation of numbers and variables. When modeling with mathematics, the equations must reflect the physical situation. By linking the mathematics to the physical system or scenario, one can infer information without experimentation.

This activity can also be constructed as a challenge to the students. With one racer (or team) begins at one location, the competition is to determine which racer (or team) reaches their endpoint first.

With the students in a total of 4 groups, they can play the role of racers a 4 of the 5 hōlua locations. It is their job then to research their slide in effort to gain an edge on the competition! In their groups, they will obtain and calculate the necessary information.

Google Earth and Slope

By working through the lesson as a group lesson, the teacher should show the students how to obtain the elevation and ground length (base to base length) via Google Earth by using one of the 5 courses. The teacher may work through the mathematics for, say, Mauna Kea in the same exact way as Problem #1 below. In fact, it is recommended to use Problems #1 and #2 to motivate Problem #3, which would be the point of grouping the students. Within this problem, students find all the parameters of their groups' mountain. Students will discover the winner of a competition to reach the end point the each groups' hōlua slide. The students then must determine the speed they need to beat the current first place holder. With the exception of Problem #4, the remaining problems focus on different scenarios, using the same ideas developed in Problems #1-3, but more in a "word problem" format and can be used as a worksheet, Problem #7 being a real challenge problem!

For students with more of a trigonometric background, or are learning the subject, Problem #4 challenges students to determine the length of slide comprised of linear segments.

Answers are in red.

Discussion for Slope Problems

Upon completing the problems associated with this activity, students should (if they have not already) begin to inquire about the validity and practicality of their answers. Either via unit conversion and/or verbal justification, the students must be capable of explaining their reasoning for why they believe these solutions to be realistic or not.

Problems

For student worksheet, please see the section immediately following:

- Problem #1: There is a hōlua in Keauhou called Kane‘aka. The beginning of the slide is 4700 [ft] inland and 600 [ft] above sea level. Assume the hōlua is linear and that the endpoint is at the edge of the ocean.

  1. How long is the slide? What methods can be used to determine how long it is?
     The length can be determined a number of ways. You can use the Pythagorean theorem, the distance formula, or trigonometry .
     By Pythagorean theorem, we obtain the length of the slide is $\sqrt{4700^2 + 600^2} = 4738.14$[ft].
     Using the coordinates $(600, 4700)$ and $(0, 0)$, the distance between those two points is
     $\sqrt{(4700 - 0)^2 + (600 - 0)^2} = 4738.14$[ft].

  2. What is the angle of elevation of the slide?
     From trigonometry, we know can find the angle of elevation via:
     $\tan(\theta) = \frac{600}{4700} \implies \theta = \arctan(\frac{600}{4700})$ and find that $\theta \approx 7.275°$.
     Then use either sine or cosine by the following:
     $\sin(7.275) = \frac{600}{c} \implies c = \frac{600}{\sin(7.275)}$
     $\cos(7.275) = \frac{4700}{c} \implies c = \frac{4700}{\cos(7.275)}$
     both of which yield 4738.14 [ft].

- Problem #2: A racer is sliding down Kane‘aka. His coach has marked a 300 foot section with flags. When the racer passes the first flag, the coach begins a timer. After 8 seconds have passed, the racer passes the second flag. What is the average velocity of the racer?

  The average velocity, $\bar{v} = \frac{\text{change in distance}}{\text{change in time}} = \frac{\Delta x}{\Delta t} = \frac{300}{8} = 37.5$ [ft/s].

- Problem #3: Pele loved he‘e hōlua so much that several chiefs built a fabled hōlua track from the peak of every mountain on Hawai‘i all the way to the ocean in her honor. Assume that they are all linear. Each ended at the location given in the table.

Location	Endpoint	Elevation [ft]	Distance [ft]	Length [ft]	Equation	Angle of Elevation[°]
Mauna Kea	Hilo					
Mauna Loa	Ka Lae					
Hualālai	Kona					
Kīlauea	Halape					
Kohala	Kawaihae	.				

Looking up elevation data online (source: `www.wikipedia.com`), then using Google Earth to measure the distances from the summit to the endpoint,

1. Find the length of each slide.

2. Write equations for each of the hōlua.
3. Determine the angle of each respective slide.

The following table lists the approximate values:

Location	Endpoint	Elevation[ft]	Distance[ft]	Length[ft]	Equation	Angle°
Mauna Kea	Hilo	13796	141300	142470	$y = \frac{9.7}{100}x$	5.57
Mauna Loa	Ka Lae	13679	211830	212271	$y = \frac{6.5}{100}x$	3.69
Hualālai	Kona	8271	152200	152425	$y = \frac{5.4}{100}x$	3.11
Kīlauea	Halape	4190	55988	56145	$y = \frac{7.5}{100}x$	4.28
Kohala	Kawaihae	5480	46300	46623 8	$y = \frac{11.8}{100}x$	6.75

4. Imagine that competitors began sliding down the hōlua with the velocities given in the table.

Location	Endpoint	Initial Velocity [ft/s]	Length [ft]	Travel Time $t = \frac{d}{r}$ [s]
Mauna Kea	Hilo	100	142470	1425
Mauna Loa	Ka Lae	150	212271	1335
Hualālai	Kona	40	152425	3811
Kīlauea	Halape	45	56145	1248
Kohala	Kawaihae	30	46623	1554

(a) In what order would the competitors finish? The racers from each individual mountain would finish in this order:

i. Kīlauea at 1248 [s]
ii. Mauna Loa at 1335 [s]
iii. Mauna Kea at 1425 [s]
iv. Kohala at 1554 [s]
v. Hualālai at 3811 [s]

(b) What would be the split times (i.e. the difference in times between competitor #1 and #2, between #2 and #3, etc.)?

1st place finished 87 [s] ahead of 2nd place, 2nd place finished 90 [s] ahead of 3rd place, 3rd place finished 129 [s] ahead of 4th place, and 4th place finished 2257 [s] ahead of 5th place.

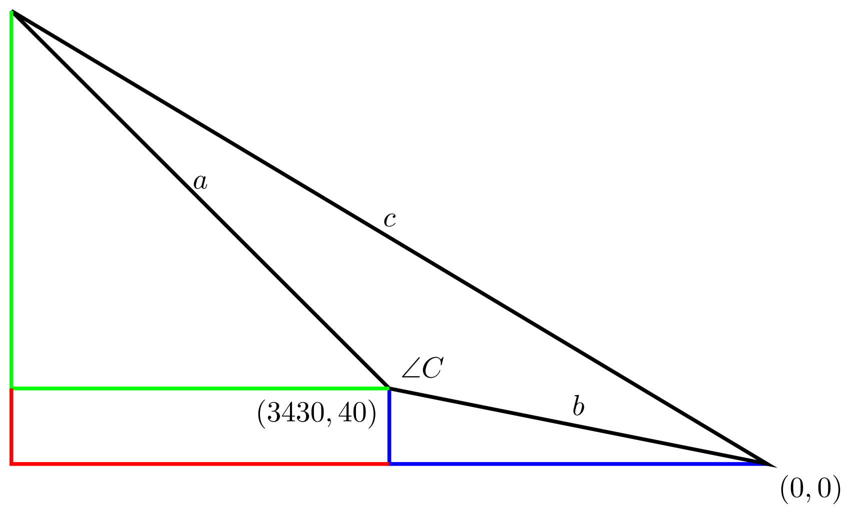
(c) At what speed (respectively) would each competitor need to travel at in order to beat competitor #1?

All racers would have to finish at least as fast as the Kīlauea racer, so at their respective path lengths, they each must travel the following rates:

i. Mauna Loa would have to travel $\frac{212271}{1248} \approx 170$[ft/s].
ii. Mauna Kea would have to travel $\frac{142470}{1248} \approx 114$[ft/s].
iii. Kohala would have to travel $\frac{46623}{1248} \approx 37$[ft/s].

iv. Hualālai would have to travel $\frac{152425}{1248} \approx 122$[ft/s].

- Problem #4: Assume that the hōlua called Kane'aka is not directly linear. Instead, it is comprised of two linear pieces at different angles.



1. What is the angle of elevation of the first linear section of the track? What is the angle of elevation of the second linear section of the track? What is the rationale for designing a hōlua like this?
   The angle of each respective section is calculated as follows:

$$\tan(\theta_1) = \frac{\Delta y}{\Delta x} = \frac{600-40}{4700-3430}$$
$$\theta_1 = \arctan\left(\frac{560}{1270}\right) \approx 23.8°$$
$$\tan(\theta_2) = \frac{\Delta y}{\Delta x} = \frac{40-0}{3430-0}$$
$$\theta_2 = \arctan\left(\frac{40}{3430}\right) \approx 1°$$

   A slide would be designed this way in order that the racer could come to a stop or slow significantly before the end of the track.

2. What are the lengths of each linear section respectively?
   Using the distance formula:

$$a = \sqrt{(600-40)^2 + (4700-3430)^2} \approx 1388[ft]$$
$$b = \sqrt{(40-0)^2 + (3430-0)^2} \approx 3431[ft]$$

3. Using the Law of Cosines (which is the general version of the Pythagorean Theorem), given by $c^2 = a^2 + b^2 - 2ab \cdot \cos(\angle C)$, determine the length of the slide if there was not a "bend" in it. The

angle between the two linear sections of the track is $\angle C = 157.2°$.
Plugging in the values from the previous problem:

$$c^2 = a^2 + b^2 - 2ab \cdot \cos(\angle C)$$
$$c^2 = 1388^2 + 3431^2 - 2 \cdot 1388 \cdot 3431 \cdot \cos(157.2°)$$
$$c = 4741.2[ft]$$

- Problem #5: In ancient time, participants of he'e hōlua would compete against participants of he'e nalu. As soon as a surfer would begin to paddle for a wave, a slider would begin running toward the hōlua. These racers would compete to see who could reach a ki'i marker on the beach first. Assume a surfer is 1000 [ft] offshore and catches a wave traveling 30 [ft/s]. In the same instant, a hōlua racer begins down Kane'aka (assume it is linear) at a rate of 140 [ft/s].

  1. Which competitor will reach the ki'i first?
     $t_{\text{he'e nalu}} = \frac{1000}{30} \approx 33.\bar{3}$[s].
     $t_{\text{he'e hō}lua} = \frac{4738}{140} \approx 33.84$[s].
     The surfer wins the race.
  2. How long after the first competitor arrives does the other competitor arrive?
     The hōlua racer comes in second, only losing by $33.84 - 33.\bar{3} \approx 0.51$[s] to the surfer.
  3. How fast will the other competitor need to travel in order to win in a second race under similar conditions?
     The hōlua racer would have to arrive at the ki'i before the time of the surfer, hence within the same time. The hōlua racer must travel at a rate $r = \frac{4738}{33.\bar{3}} \approx 142.12$[ft/s].

- Problem #6: Kanawali, a chief from Puna who was well known for his skill at hōlua, was challenged to a race by Pele herself (in disguise, no less). Kanawali was winning very easily, which angered Pele. She conjured a fast moving lava flow which she rode in order to destroy Kanawali. Assume Kanawali was 150 [ft] ahead and traveling at 90 [ft/s]. Kanawali is only 1000 [ft] from the end of the track. What is the minimum speed Pele must travel in order to destroy Kanawali before he escapes?

  The amount of time that Kanawali will take to reach the end of the slide is $t = \frac{1000}{90} \approx 11.\bar{1}$[s].
  In order to catch Kanawali, Pele must travel the 1000 feet + 150 feet that she is behind Kanawali.
  Thus, Pele must travel at least as fast as $r = \frac{1150}{11.\bar{1}} \approx 103.5$[ft/s].

- Problem #7: Assume that Kane'aka is a linear slide. While preparing for a race, the competitors ran out of coconut oil to lubricate the track. Only the first 40% of the track was lubricated using this oil. On this section, competitors travel at a rate of 70 [ft/s]. The remaining 60% was left unlubricated. On this section, competitors travel at a rate of 40 [ft/s].

  1. How long does it take to travel from the top of the slide to the bottom?
     First, the length of Kane'aka, from Problem #1, is $d = 4738$[ft].

     The length of the unlubricated section is $d_1 = 0.6 \cdot 4738 = 2842.8$[ft].
     The length of the lubricated section is $d_2 = 4738 - 2842.8 = 0.4 \cdot 4738 = 1895.2$[ft].

The time to travel the unlubricated section is $t_1 = \frac{2842.8}{40} \approx 71.07$[s].
The time to travel the lubricated section is $t_2 = \frac{1895.2}{70} \approx 27.07$[s].
Hence, the total time to transverse Kane‘aka is $T = t_1 + t_2 = 71.07 + 27.07 \approx 98.14$[s].

2. One of the competitors shows up with his own coconut oil which he uses to lubricate another 30% of the unlubricated track. Now how long does it take to travel the length of the hōlua?

Again, the length of Kane‘aka, from Problem #1, is $d = 4738$[ft].
The length of the unlubricated section is now $d_1 = 0.3 \cdot 4738 = 1421.4$[ft].
The length of the lubricated section is now $d_2 = 4738 - 1421.4 = 0.7 \cdot 4738 = 3316.6$[ft].
The time to travel the unlubricated section is $t_1 = \frac{1421.4}{40} \approx 35.54$[s].
The time to travel the lubricated section is $t_2 = \frac{3316.6}{70} \approx 47.38$[s].
Hence, the total time to transverse Kane‘aka is $T = t_1 + t_2 = 35.54 + 47.38 \approx 82.92$[s].

He‘e Hōlua Worksheet **Name:** ______________________________

Problems

- Problem #1: There is a hōlua in Keauhou called Kane‘aka. The beginning of the slide is 4700 [ft] inland and 600 [ft] above sea level. Assume the hōlua is linear and that the endpoint is at the edge of the ocean.

  1. How long is the slide? What methods can be used to determine how long it is?

  2. What is the angle of elevation of the slide?

- Problem #2: A racer is sliding down Kane‘aka. His coach has marked a 300 foot section with flags. When the racer passes the first flag, the coach begins a timer. After 8 seconds have passed, the racer passes the second flag. What is the average velocity of the racer?

- Problem #3: Pele loved he'e hōlua so much that several chiefs built a fabled hōlua track from the peak of every mountain on Hawai'i all the way to the ocean in her honor. Assume that they are all linear. Each ended at the location given in the table.

  Looking up elevation data online (source: `www.wikipedia.com`), then using Google Earth to measure the distances from the summit to the endpoint,

  a) Find the length of each slide.
  b) Write equations for each of the hōlua.
  c) Determine the angle of each respective slide.

Location	Endpoint	Elevation [ft]	Distance [ft]	Length [ft]	Equation	Angle of Elevation[°]
Mauna Kea	Hilo					
Mauna Loa	Ka Lae					
Hualālai	Kona					
Kīlauea	Halape					
Kohala	Kawaihae	.				

  d) Imagine that competitors began sliding down the hōlua with the velocities given in the table.

Location	Endpoint	Initial Velocity [ft/s]	Length [ft]	Travel Time $t = \frac{d}{r}$ [s]
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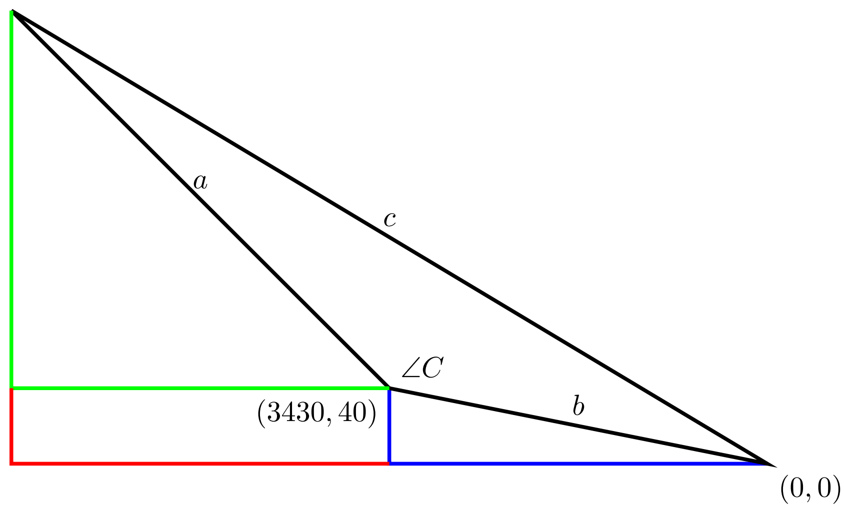
    i) In what order would the competitors finish?

    ii) What would be the split times (i.e. the difference in times between competitor #1 and #2, between #2 and #3, etc.)?

    iii) At what speed (respectively) would each competitor need to travel at in order to beat competitor #1?

- Problem #4: Assume that the hōlua called Kane‘aka is not directly linear. Instead, it is comprised of two linear pieces at different angles.



1. What is the angle of elevation of the first linear section of the track? What is the angle of elevation of the second linear section of the track? What is the rationale for designing a hōlua like this?

2. What are the lengths of each linear section respectively?

3. Using the Law of Cosines (which is the general version of the Pythagorean Theorem), given by $c^2 = a^2 + b^2 - 2ab \cdot \cos(\angle C)$, determine the length of the slide if there was not a "bend" in it. The angle between the two linear sections of the track is $\angle C = 157.2°$.

- Problem #5: In ancient time, participants of he'e hōlua would compete against participants of he‘e nalu. As soon as a surfer would begin to paddle for a wave, a slider would begin running toward the hōlua. These racers would compete to see who could reach a ki‘i marker on the beach first. Assume a surfer is 1000 [ft] offshore and catches a wave traveling 30 [ft/s]. In the same instant, a hōlua racer begins down Kane‘aka (assume it is linear) at a rate of 140 [ft/s].

1. Which competitor will reach the ki‘i first?

2. How long after the first competitor arrives does the other competitor arrive?

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- Problem #6: Kanawali, a chief from Puna who was well known for his skill at hōlua, was challenged to a race by Pele herself (in disguise, no less). Kanawali was winning very easily, which angered Pele. She conjured a fast moving lava flow which she rode in order to destroy Kanawali. Assume Kanawali was 150 [ft] ahead and traveling at 90 [ft/s]. Kanawali is only 1000 [ft] from the end of the track. What is the minimum speed Pele must travel in order to destroy Kanawali before he escapes?

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  1. How long does it take to travel from the top of the slide to the bottom?

  2. One of the competitors shows up with his own coconut oil which he uses to lubricate another 30% of the unlubricated track. Now how long does it take to travel the length of the hōlua?

Free-Body Construction

Time	Procedure
5 mins	Historical Introduction & Motivation of the Mathematics
20 mins	Construct the free-body diagram
45 mins	Forces Discussion
20 mins	Problems

Historical introduction and motivation of the mathematics

Using the introduction above, share with the students the Hawaiian history of the heʻe hōlua. Explain to the students how mathematics is used to model actual physical situations, as this is precisely what they will do in this activity. It is important for students to know in addition that mathematics is more than simply manipulation of numbers and variables. When modeling with mathematics, the equations must reflect the physical situation. By linking the mathematics to the physical system or scenario, one can infer information without experimentation.

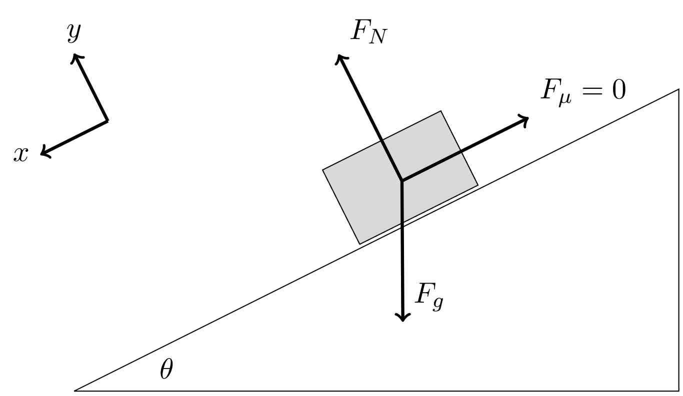
For example, $F = ma$ models the equation for Newtonian force. From this equation, we can deduce several traits that are true for Newtonian forces:

- with acceleration fixed (a is constant), the force F is a function of mass, m. If m increases, so does F. If mass decreases, so does the force.
- with fixed mass (m is constant), the force is a function of acceleration. If a increases, so does F. If acceleration decreases, so does the force.

This makes perfect sense right? Slow down a car and, if there is an impact, the force of the car impacting an object will be diminished! With this kind of motivation, the students can have faith in the mathematics.

Construct the free-body diagram

A free-body diagram is an image depicting all the forces acting on the system in question. For the heʻe hōlua, the diagram looks like:



Forces Discussion

Below you will find the discussion questions. The purpose of these questions are to guide the students through the construction of the diagram, but more importantly the interpretation of the diagram. They begin with simple interpretations of the model and work up to having the student determine specific values using trigonometric concepts and interpreting/manipulating equations to draw conclusions about the system. Ultimately, this leads to the concept of rates of change, which is tied to differentiation and integration. Depending on the level of the class, different mathematical techniques should be used.

Answers are in red.

Problems

Below you will find a list of problems. Use these problems to test the students' understanding of the material. In addition, upon obtainment of the answers, the quantities and equations should be discussed. In particular, students need to check on the practicality/realism of the solutions. For example, how fast is 0.35 m/sec^2 in a context the students can understand? Is this speed reasonable? Questioning and interpreting the values, the equations, and its parameters (coefficients, inputs, and outputs) is equally as important as a correct solution.

Answers are in red.

Forces Discussion

Note: In the discussion we disregard units.

a) Using arrows, show the direction of the different forces acting on the sled in the following free body diagram.

   Include the following forces: Gravity, Friction, Normal

b) Notice the orientation of the axes in the diagram. Why was this orientation chosen or does the orientation matter?

   This orientation is not mandatory, but it was chosen so that the object's acceleration is purely in the x direction

c) Given an angle of inclination, θ, what is the component of the force of gravity acting on the sled in the x direction? And in the y direction?

   x direction: $F_G \sin(\theta)$, y direction: $F_G \cos(\theta)$

d) Using Newton's Second Law of Motion, which states that the sum of the forces acting on an object is equivalent to the object's acceleration multiplied by its mass ($\sum_i F_i = m \cdot a$), what is an equation for the sled's acceleration in the x direction? And in the y direction? (Assume the force of gravity is given by $m \cdot g$ where m is the mass of the object and g is a constant. Also assume the force of friction is given by $F_N \cdot \mu$ where F_N is the normal force and μ is the coefficient of friction.)

   x direction: $ma_x = mg\sin(\theta) - \mu F_N$, y direction $ma_y = F_N - mg\cos(\theta)$

e) Simplify your results in the previous question as much as possible. Note that we are only interested in the sled's acceleration in the x direction. (What is the sled's acceleration in the y direction?)

x direction: $a_x = g\sin(\theta) - \mu g\cos(\theta)$. There is no acceleration in the y direction.

f) Does our final equation depend on mass? Discuss.

It does not because our acceleration only depends on the force of gravity.

g) What are some reasons why the sled's acceleration may not be constant?

The angle of inclination may not be constant as well as the coefficient of friction.

h) Recall that an object's average acceleration over a given time period is its change in velocity, Δv, divided by the change in time, Δt. $\left(\text{i.e. } a_{average} = \frac{\Delta v}{\Delta t}.\right)$

Taking a limit as Δt approaches 0 we can obtain instantaneous acceleration. We immediately notice that this is the usual definition of a derivative of the velocity function with respect to time. Therefore $a_{instantaneous} = \frac{dv}{dt}$ where $\frac{d}{dt}$ is the usual differential operator. Similarly we get that instantaneous velocity is the derivative of the displacement function with respect to time. $\left(\text{i.e. } v_{instantaneous} = \frac{ds}{dt}\right)$.

Notice that using Newton's Second Law gave us a function of acceleration with respect to time. And so, we can antidifferentiate with respect to time to obtain functions for our sled's velocity and displacement.

Problems

Solve the following problems using the theory developed in the discussion. Assume that the constant of gravity is given by $g = 9.8$ meters/second^2, the coefficient of friction is constant given by $\mu = .125$, and the sled has no initial velocity.

There is a he'e hōlua site in Keauhou on the side of Hualālai called Kane'aka that originally measured about $1,524$ meters long.

1. From the image provided, what would be a reasonable angle of inclination if we wanted a constant value?(Note that the image is measured in feet).

$.16$ radians

2. What is the sled's acceleration in the x direction as a function of time?

$.35$ m/sec^2

3. Write equations for the sled's instantaneous velocity and displacement as functions of time.

$v(t) = .35t$, $s(t) = \frac{.35}{2}t^2$

4. What is the sled's velocity and displacement after 1 minute?

$v(60) = 21$m/sec, $s(60) = 630$ m

Common Core Standards

Grade 7:

Use properties of operations to generate equivalent expressions.

- EE1. -Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

- EE.3 - Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically
- EE.4 - Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Grade 8:

Work with radicals and integer exponents

- EE.4 - Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology

Understand the connections between proportional relationships, lines, and linear equations.

- EE.6 - Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b.

Use functions to model relationships between quantities

- F.4 - Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

Understand and apply the Pythagorean Theorem

- G.7 -Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
- G.8 - Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

High School:

Reason quantitatively and use units to solve problems.

- N-Q.1 - Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

- N-Q.2 - Define appropriate quantities for the purpose of descriptive modeling.

Represent and model with vector quantities

- N-VM.1 - Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, $|v|$, $\|v\|$).
- N-VM.3 - Solve problems involving velocity and other quantities that can be represented by vectors.

Create equations that describe numbers or relationships

- A-CED.1 - Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
- A-CED.2 - Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- A-CED.3 - Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.
- A-CED.4 - Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Interpret the structure of expressions

- A-SSE.1 - Interpret expressions that represent a quantity in terms of its context.

Understand solving equations as a process of reasoning and explain the reasoning

- A-REI.1 - Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Solve equations and inequalities in one variable

- A-REI.3 - Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
- A-REI.4 - Solve quadratic equations in one variable.

Define trigonometric ratios and solve problems involving right triangles

- G-SRT.7 - Explain and use the relationship between the sine and cosine of complementary angles.

- A-REI.6 - Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Understand the concept of a function and use function notation

- F-IF.2 - Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Interpret functions that arise in applications in terms of the context

- F-IF.4 - For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- F-IF.5 - Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- F-IF.6 - Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval

Build a function that models a relationship between two quantities

- F-BF.1 - Write a function that describes a relationship between two quantities.

Construct and compare linear, quadratic, and exponential models and solve problems

- F-LE.1 - Distinguish between situations that can be modeled with linear functions and with exponential functions.

Interpret expressions for functions in terms of the situation they model

- F-LE.5 - Interpret the parameters in a linear or exponential function in terms of a context.

Apply geometric concepts in modeling situations

- G-MG.3 - Apply geometric methods to solve design problems