Heiau

Introduction

In ancient Hawaiʻi, heiau were a major part of Hawaiian life. There were two major orders of heiau: (1) agricultural or economy related ones which were dedicated to Lono offerings of pigs, vegetables, and bark cloth were given to guarantee rain and (2) agricultural fertility and plenty. The others were large sacrificial government war temples, luakini (heiau po'okanaka), upon whose altars human lives were taken when assurance of success in combat was requested or when a very grave state emergency, such as pestilence or famine, dictated that the highest religious authority, Ku, be approached for help.

Building these mammoth monuments was the responsibility of the aliʻi, due to the fact that only they could command the necessary resources to build them, maintain the priests, and to secure the sacrifices that were required for the rituals. Heiau were usually built in or near villages, on prominent hills or ridges, on cliffs with a good view of the sea, or on plateaus between the coast and the mountains. After the arrival of the high priest Paʻao around the thirteenth century, heiau built on these hillsides were enclosed with high stone walls, preventing the masses from participating as freely in the worship ceremonies.

Heiau sometimes had other structures built within them and were often constructed using the wood of ōhiʻa lehua, thatched pili grass, and the cordage was woven from olonā. Large figures, carved from the wood of ōhiʻa lehua as well, that represented gods, called kiʻi, were placed in and around the heiau. Other structures built upon the ancient heiau were small houses used for a particular purpose and an ʻanuʻu or oracle tower. Gathering materials for these heiau was an impressive achievement in and of itself. The stones used for the Puʻukoholā Heiau were passed hand-to-hand all the way to the site via a 20 mile long human chain.

Grade Levels

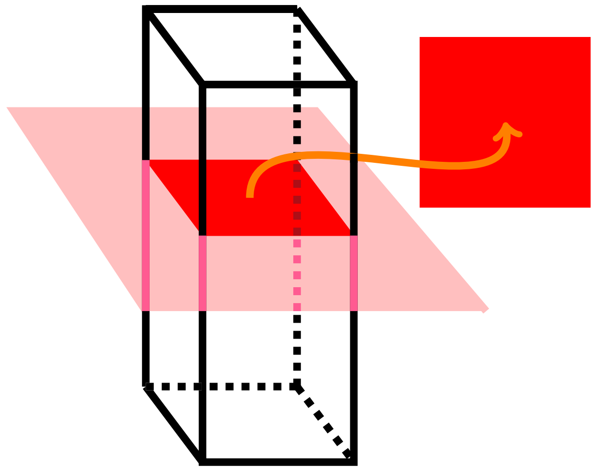
This activity is intended for students in grades 7–12. The primary focus of this activity is on geometric solids, specifically 3D composite solids and cross sections. For high school juniors and seniors, this activity can be extended to include concepts in statistics.

Mathematical Topics

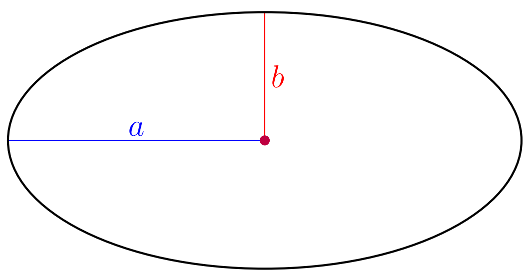
- **Geometry**:
  - **Geometric Solids**: The geometry of 3-dimensional space is called *S*olid Geometry. Common 3-dimensional shapes include sphere, cube, rectangular prism, cone, and pyramid; however, this lesson will not look at spherical or conical shapes.
  - **3-Dimensional Composite Figures:** Three dimensional composite figures are solids whose composition consists of simpler solids (e.g., cubes, spheres, hemispheres, pyramids, cones, etc...) or

portions thereof. This activity's composite figures primarily consist of rectangular and triangular prisms.

- **Cross sections:** A surface that is or would be exposed by slicing a plane through a solid. Here is one example of the cross section of a rectangular prism with a square base:



- **Perimeter of an ellipse:** The perimeter of an ellipse is not as straight forward as other perimeter formulas. Without calculus, the best you can obtain is an approximation. The formula for the perimeter P of the following ellipse where a is the major axis and b is the minor axis is:



$$P \approx 2\pi\sqrt{\frac{a^2+b^2}{2}}$$ or

$$P \approx \pi\left[3(a+b) - \sqrt{(3a+b)(a+3b)}\right].$$

- **Statistics:**
  - **Confidence intervals:** A range of values so defined that there is a specified probability that the value of a parameter lies within it. In this lesson confidence intervals are used to determine the volume that rocks used for building heiau take up. A $100(1-\alpha)\%$ confidence interval for the mean. Note that a 100% confidence interval is useless as a 100% confidence interval spans all of the real numbers.

Materials and Resources

- Scratch paper for calculations and paper for students to draw their diagrams on
- Modeling clay or play-dough (use Ziploc bags to keep clay from drying out)

- Wax Paper
- Ruler
- String to cut clay with

Activity: Building Your Heiau Efficiently

As mentioned previously, constructing a heiau was a tremendous feat with Hawaiians having to carrying large stones over long distances. The purpose of this activity is to have the students rely on their mathematical knowledge to efficiently build their own heiau.

1. Students will get into groups of 3 or 4 and create a diagram of a heiau they would like to design. Their diagrams must include dimensions and cross sections.
2. With the diagrams in hand, the students then must calculate the volume of their heiau.
3. The students must now scale down their design in order to create a clay model of their heiau.
4. Before the students begin building their heiau model, first ask them to calculate how much clay they need in order to create an accurate scale model.
5. Have the the students discuss and record ways in which they could avoid error in building their model (i.e. Use measurement tools, etc...).
6. Once the planning process is complete, students may begin building their scale model heiau.
7. While constructing, require the students to record the amount of clay they are using. Ultimately, the students will determine the percentage error of clay initially deemed required versus the amount actually used (theoretical vs. actual).
8. Have them discuss as a group why this error has occurred and ways in which they could have reduced the error.
9. Lastly, have each group present to the class their model, how they attempted to maintain the accuracy of their clay model diagram, and how they may reduce the error if they were to repeat the process.

Activity: Statistics Extentsion

When extending this activity to students with a statistic background, the following items may take the respective place of the previous instructions.

2–6. Have students will create a number of clay/playdough stones (20-40) and create an experiment to calculate a 95% confidence interval for the number of clay stones they will need in order to create their model.

7. When students begin to build their model heiau, instruct them to record the total number of stones added to the heiau.
8. When completed, ask the students if the number of clay stones they used was in their confidence interval? How are ways in which their experiment could have been improved?

Notes

Geometry

Discuss with the students the concepts behind cross sections of a three dimensional shape. In particular, emphasize that the volume of a regular solid is the area of that cross section multiplied by the height of the solid (or width/length) in which the cross section spans.

Statistics

Confidence intervals are used for a variety of real life scenarios including determination of the number of beds a hospital necessary in emergency situations, the amount of food required to feed a large group of people, and the amount of non-uniform sized construction materials may be needed for a construct.

In regards to the heiau, ali‘i would tax the people through resources and labor in order to build a new heiau. If he were to tax too much, the people might object, where as if he taxed too little the heiau would not be completed. Thus, it was critical to have an accurate estimate of the materials required. I histories times, the ali‘i used their intuition to determine the amount of people and resources he would need to construct a heiau. For this activity, we will use confidence intervals to determine a similar estimate.

- A confidence interval for the average value or mean of a random variable is an interval estimate of a population parameter. In this case, a confidence interval is the mean number of rocks or volume. Before calculating a confidence interval, first an experiment must be conducted in order to gather data. Calculating a 95% confidence interval for the mean is to show there is a probability of 95% that running an experiment (randomly selecting stones) will result in falling within a determined upper and lower bound. For example, to say we have a 95% confidence interval of the size of rocks used to build a heiau means that if we randomly choose a rock, there is a 95% chance that it will be within our experimentally determined range of acceptable rock sizes. ***Note***: if the size of the confidence percentage increases, there is an increase in the size of the interval. For example, a 100% confidence interval would be $(-\infty, \infty)$; this says that you are 100% confident that the average is going to be a real number, which does not help in application.
- **Recall:** The formula for a $(100 - \alpha)\%$ confidence interval for the mean of a student's t-distribution is:

$$\left(\bar{x} - \frac{\sigma}{\sqrt{n}} \cdot t_{\alpha/2,n-1}, \bar{x} + \frac{\sigma}{\sqrt{n}} \cdot t_{\alpha/2,n-1}\right)$$

  where $\bar{x}$ is the sample mean, σ is the sample variance, n is the sample size, and $t_{\alpha/2,n-1}$ is the result of a one tailed t-test.

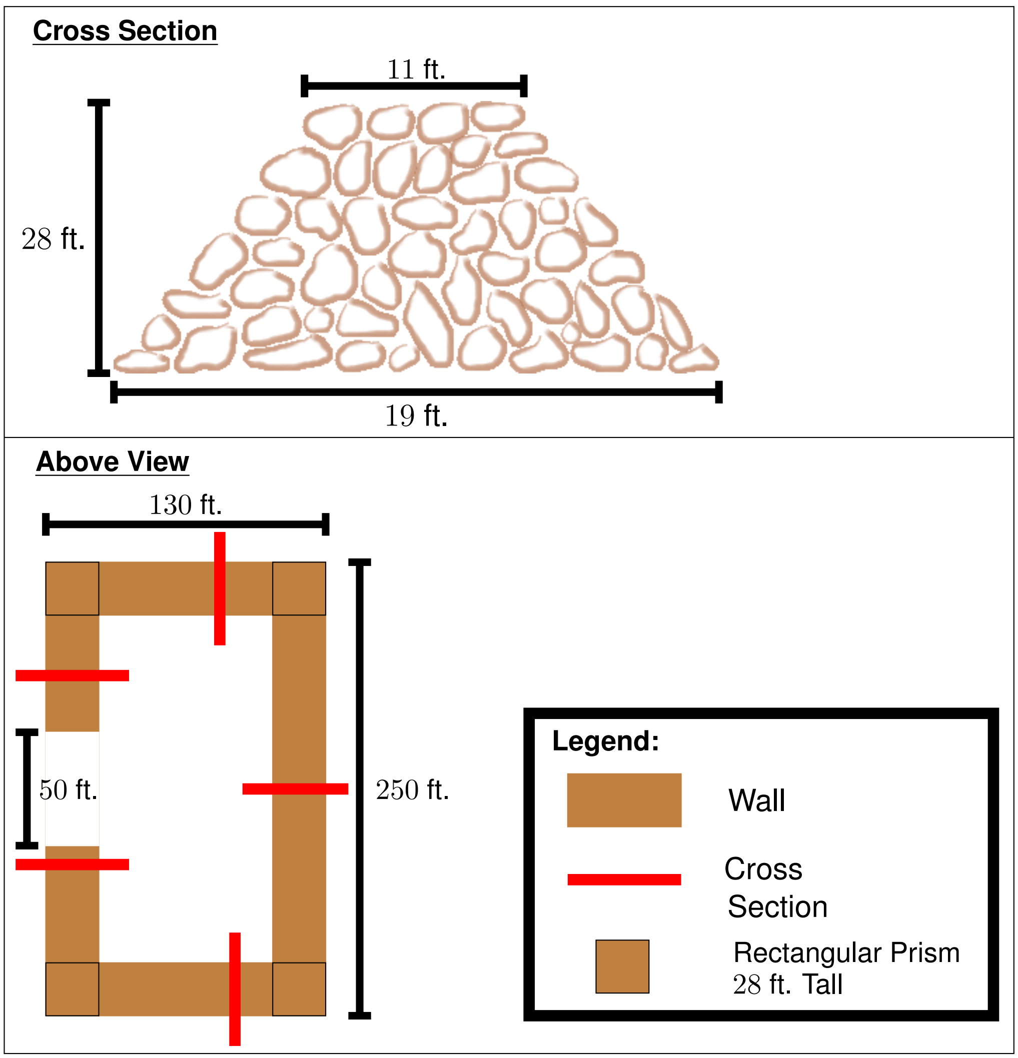
Worksheets

The following worksheets provide the students with practice of the mathematical concepts behind the construction of the heiau. It is up to your discretion whether to assign these before or after the heiau activity if at all.

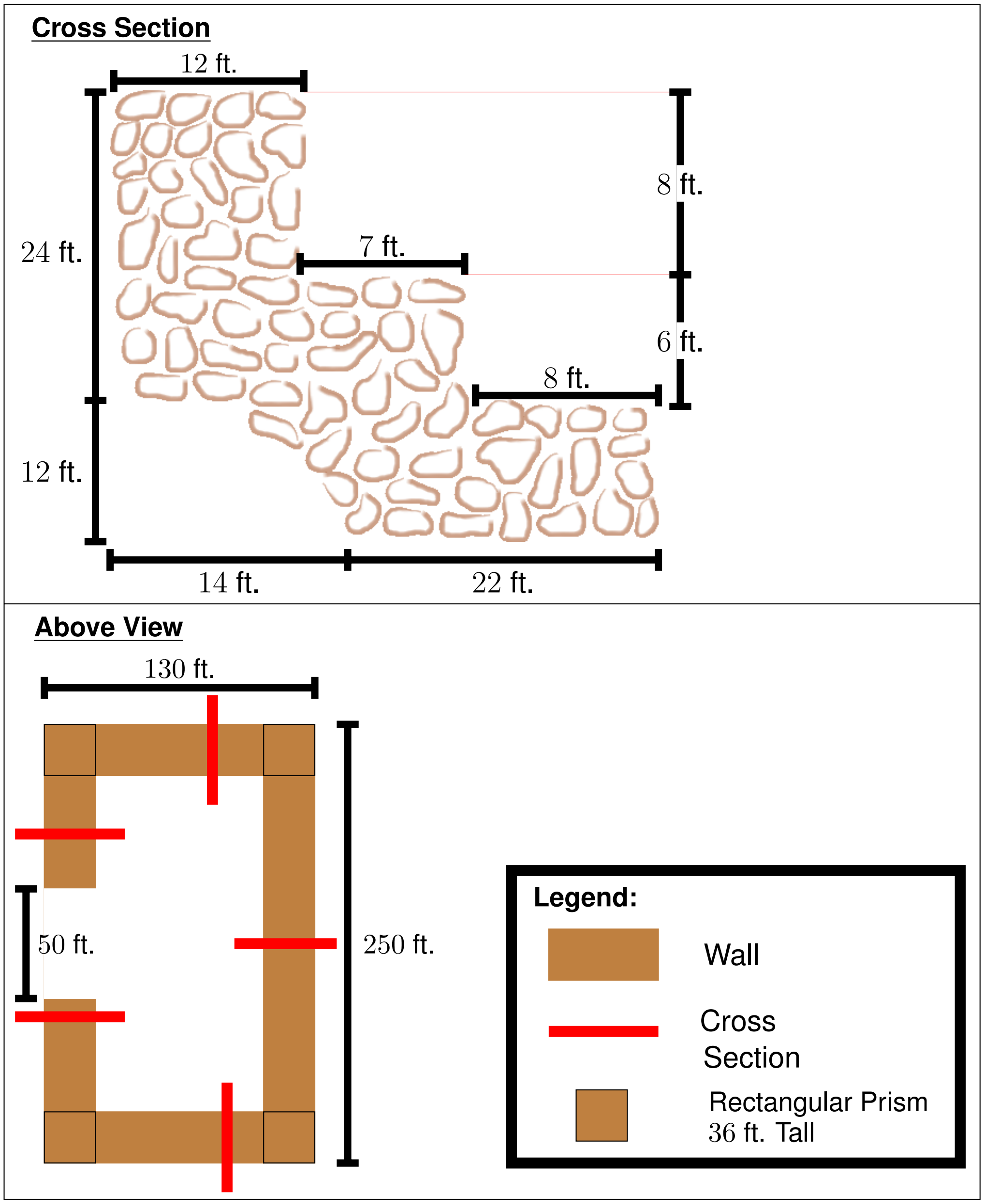
Heiau Geometry Problems **Name:** ______________________________

1. In the thirteenth century, Pāʻao, a high priest from Tahiti, came to Hawaiʻi and implemented a number of changes to the way heiau were built and used for. One of his changes was the implementation of temple courtyards to be enclosed by stone walls. Assuming that it takes roughly 12 stones to build one cubic foot of wall, calculate the number of stones required to build the following walls.

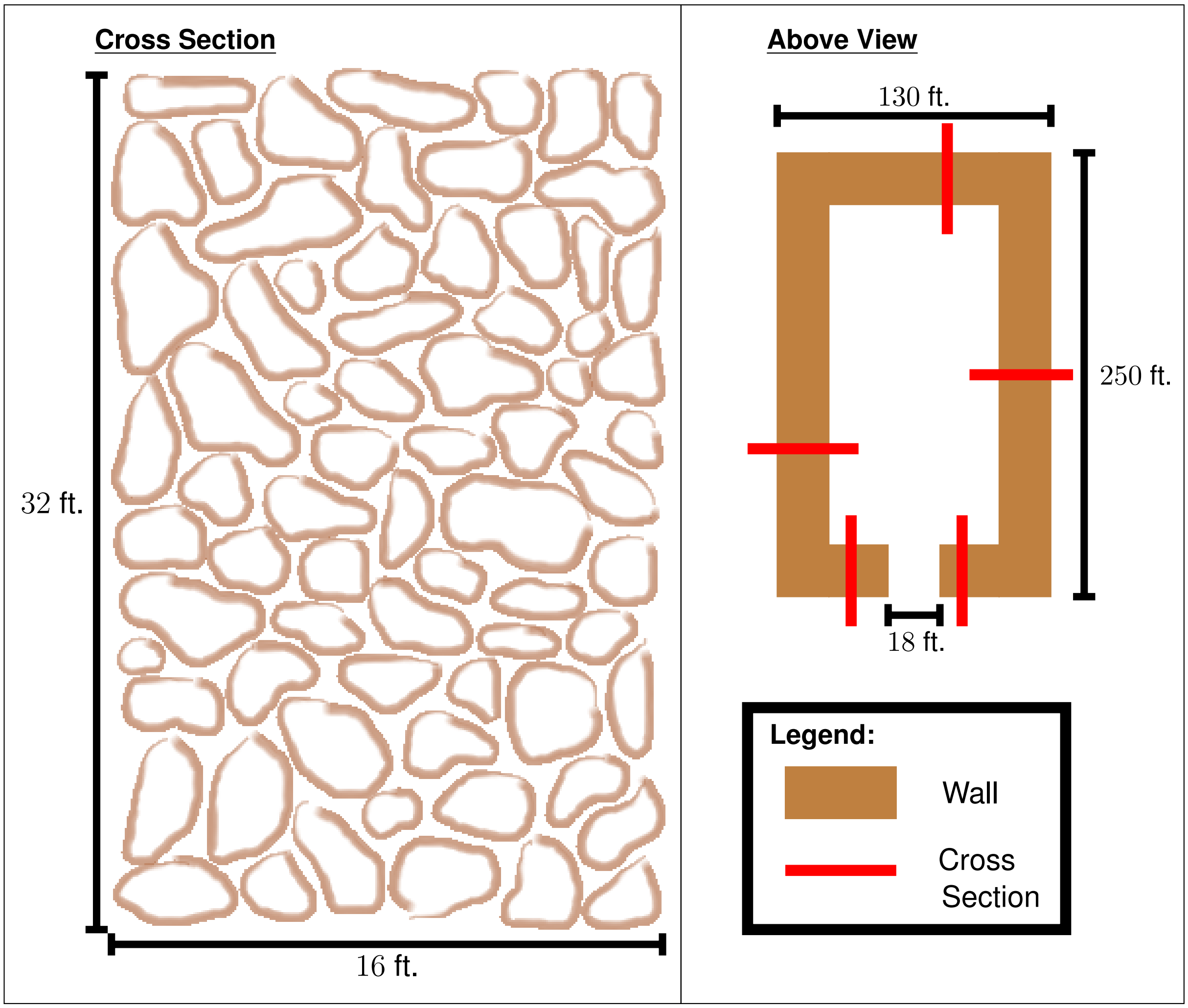
   (a)



(b)

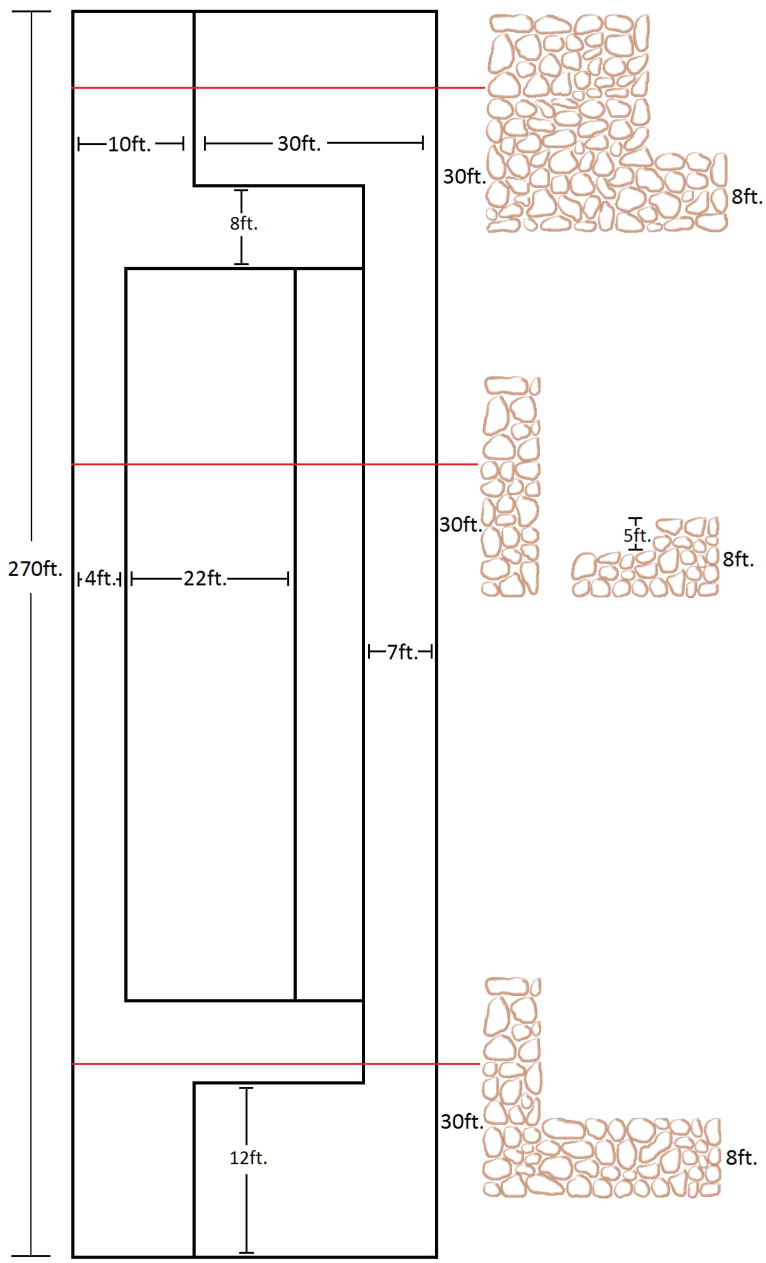

Cross Section
12 ft.
8 ft.
24 ft.
7 ft.
6 ft.
8 ft.
12 ft.
14 ft.
22 ft.
Above View
130 ft.
50 ft.
250 ft.
Legend:
Wall
Cross Section
Rectangular Prism 36 ft. Tall

(c)

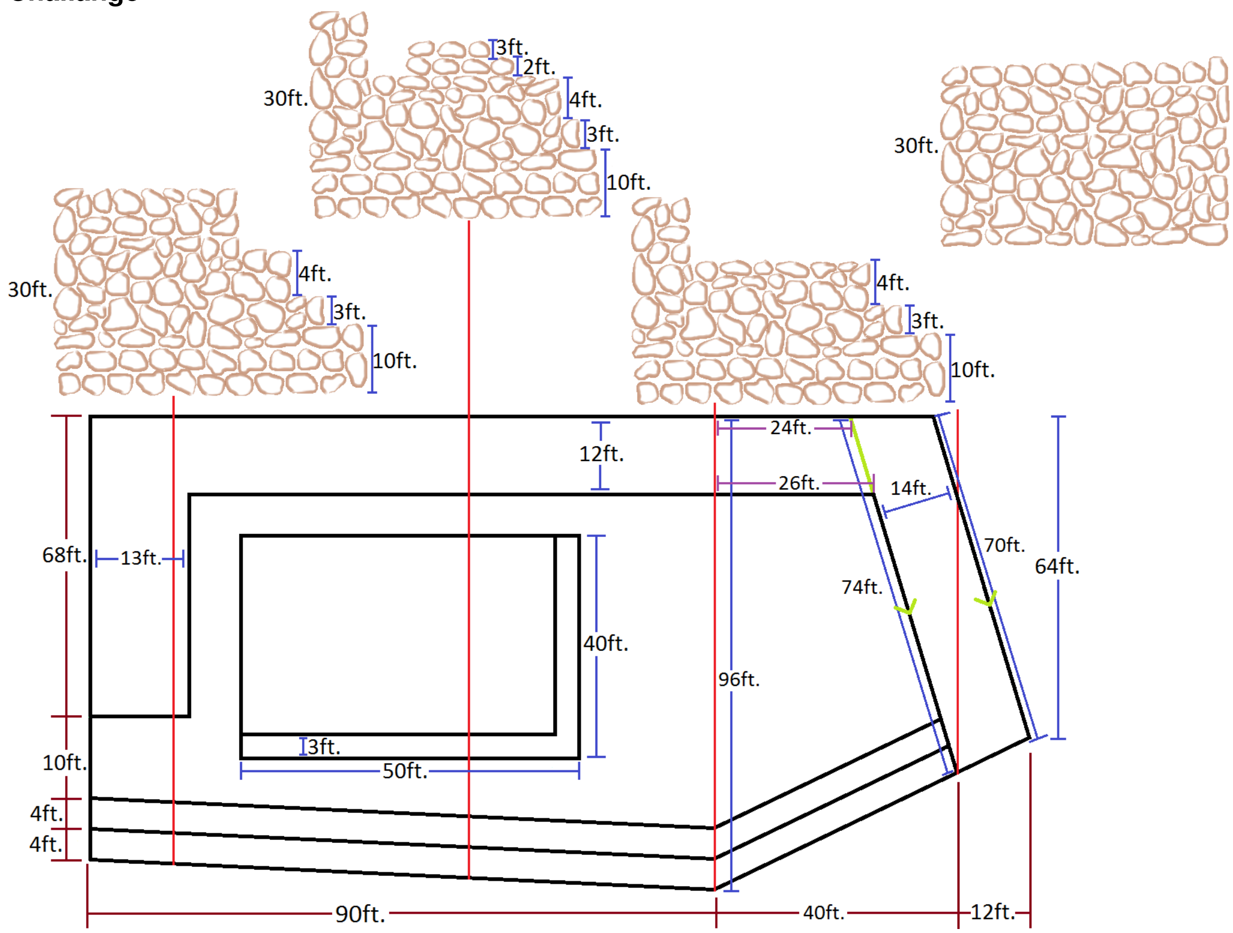

Cross Section
32 ft.
16 ft.
Above View
130 ft.
250 ft.
18 ft.
Legend:
Wall
Cross Section

2. A new heiau would first need to be modeled in the sand and then shown to the mōʻī, king, to get approved. That model would help the king to determine the tax he would lay on the people, in the form of building the new heiau. The following two diagrams are of Mailekini Heiau and Puʻukoholā Heiau respectively. Assuming that is takes roughly 74 stones to build one cubic foot of heiau, use these diagrams to calculate the volume of each heiau and the amount of rocks needed to form each heiau (the cross sections are connected to their corresponding red lines, the blue lines refer to measurements, and the black lines are the outline of the heiau where there is a change in height):

(a)



(b) **Challange**



30ft.
3ft.
2ft.
4ft.
3ft.
10ft.
30ft.
4ft.
3ft.
10ft.
4ft.
3ft.
10ft.
30ft.
24ft.
12ft.
26ft.
14ft.
68ft.
13ft.
70ft.
64ft.
74ft.
40ft.
96ft.
3ft.
10ft.
50ft.
4ft.
4ft.
90ft.
40ft.
12ft.

Heiau Statistical Problems **Name:** ______________________________

3. The following is a table on how many rocks from the Ulupo Heiau it takes to fill a space of 1 ft.^{3}. Use the following sample to create a 95% confidence interval for the average number of rocks needed to fit into a space of 1 ft.^{3}. Use this to create a 95% confidence interval for the number of stones required to create the Ulupo Heiau which is a volume of approximately $90,000$ ft.^{3}.

Sample Number	# of rocks which fit in the1 cm^3 box
1	73
2	24
3	129
4	74
5	15
6	3
7	26

4. Use the following table to create a 95% confidence interval for the mean size of the rocks used to build Ulupo Heiau.

Sample Number	Volume of rock (in^3)
1	41.3
2	2251
3	45.5
4	38.2
5	47.5
6	59.1
7	1587.5
8	2917.7
9	62.7

Is there something wrong with the result? What happened?

5. Before building a heiau the ali‘i and kahuna (priest) would have carefully picked the location for the heiau. During construction, they would over see careful placement of the stones, as to create a tight and perfect fit of all the stones. Using this information, determine which of the last two experiments would give u the most accurate confidence interval for the number of stones needed to create the heiau. Why?

Answer Key

1. (a) 359,632 ft.^{3}
   (b) $492,152 - 70,896\pi \approx 269,425.64723$
   (c) 347,136 ft.^{3}
2. (a) 103,784 ft.^{3}; $7,680,016$ stones.
   (b) $353423 - 2560\sqrt{201} \approx 317,128.73599$ ft.^{3}; $23,467,527$ stones.
3. $\left(49.143 - t_{0.025,6}\dfrac{44.771}{\sqrt{7}}, 49.143 + t_{0.025,6}\dfrac{44.771}{\sqrt{7}}\right) = (7.735, 90.551)$

   where $t_{0.025,6} = 2.447$
4. $\left(783.389 - t_{0.025,8}\dfrac{1150.64}{3}, 783.389 + t_{0.025,8}\dfrac{1150.64}{3}\right) = (-101.07, 1667.85)$

   where $t_{0.025,8} = 2.306$.

   Lower bound is negative, which is unrealistic as these bounds represent the acceptable range of the volume of rocks. This is due to the high variance amongst the samples.