Hoʻolele Lupe

Introduction

The Hawaiian demigod Maui is given credit for the invention of the hoʻolele lupe (kite). Oral traditions, as well as the creation chants of Kaelikuahulu, suggest that hoʻolele lupe, made of hau covered with kapa or pandanus leaf, were an important part of Native Hawaiian society. Not only were they flown for recreational purposes by young and old men, but they were used for maritime propulsion, fishing, and fighting as well. Hoʻolele lupe might also have held religious significance, as evidenced by the effort the missionaries used to suppress its use (Hoʻolele Lupe-An Analysis of the Ancient Practice of Hawaiian Kite-flying, Damion Sailors).

Grade Levels and Topics

- **Algebra:**
  - **Conics:** Circles, ellipses, and hyperbolas: Review the formulas for circles, ellipses, and hyperbolas with the students, you may want to remind the students how to graph each conic given its equation.
    - **Circle:**
$$(x-h)^2+(y-k)^2=r^2$$
Where r is the radius and the point (h,k) is the center of the circle.
    - **Ellipse:**
$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$
Where a is the horizontal distance from the center to the outer edge of the ellipse, b is the vertical distance from the center of the ellipse to the outer edge of the ellipse, and the point (h,k) is the center of the ellipse. If $a>b$ then a is the length of the major axis and b is the length of the minor axis and vice versa if $a<b$.
  - **Absolute value functions:** When taking the absolute value of some value $x \in \mathbb{R}$ you will end up with a positive value. This lesson does not focus on absolute value fucntions, however it is good to know for the first set of problems in the algebra section.. Common notation for the absolute value of x is $f(x)=|x|$ or $f(x)=\begin{cases} x, & \text{if } n \geq 0 \\ -x, & \text{if } n < 0 \end{cases}$
.

- **Piecewise functions:** These functions are defined by multiple subfunctions, each subfunction applies to a certain interval on the main function's domain (a subdomain).
- **Finding maxima and minima of functions:** Visually, a maxima is a hump and a minima is a dip. This lesson will focus mainly on the maxima and minima of linear functions and

Objectives

The overall purpose of this activity is to have students derive formulas and calculate areas for familiar shapes. By actually making the kites, students can use their functions to help optimize materials. This activity also offers a concrete example how to model and analyze tangible products with mathematical functions.

Contained in this PDF are worksheets for the students to practice calculating areas of shapes as well as deriving functions to describe various aspects of the kites. With limited materials, the students can use their functions to determine how to build the kite. For example, the area of a kite (the geometric shape) is $\frac{d_1 \cdot d_2}{2}$, where d_1 and d_2 are the main diagonals. Armed with the knowledge of the total area of the kite materials provided, one can determine the length of d_1 and d_2 to maximize the area of the kite!

Materials

- For a list of materials to build a kite see this website: my-best-kite.com.

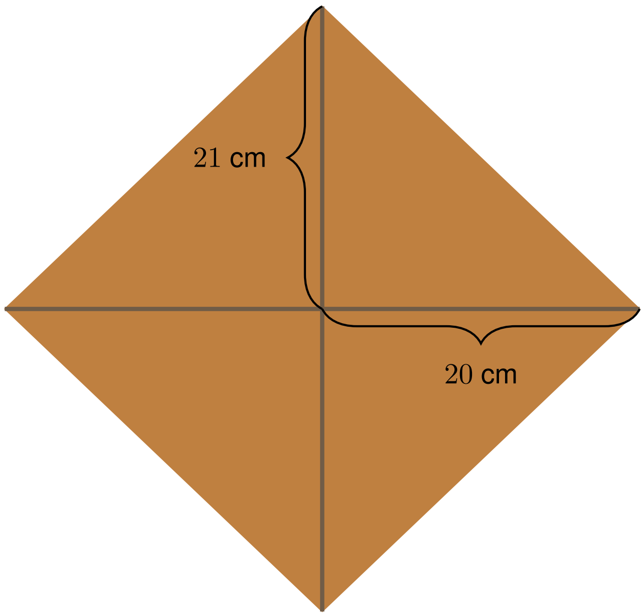
Modeling with Functions Discussion

- Facilitate a discussion about quadrilaterals.
- In this discussion, have the students articulate how a function can be used to describe quadrilaterals. Recall a functions maps from $x \to y$, with at most one y value for every x value.
- At this point, you may wish to review constant, linear, quadratic, piecewise, and absolute value functions as well as how to find the vertex, and how to find maximums and minimums of a linear function with a finite domain (not $(-\infty, \infty)$).
- Lead the discussion towards using functions to calculate characteristics of shapes such as height, perimeter, area, etc.
- Problems 3, 4, and 5, specifically use the functions created to maximize the material for a kite (geometric shape) based on the shape of the material. It is worth mentioning how knowledge of calculus simplifies this matter.

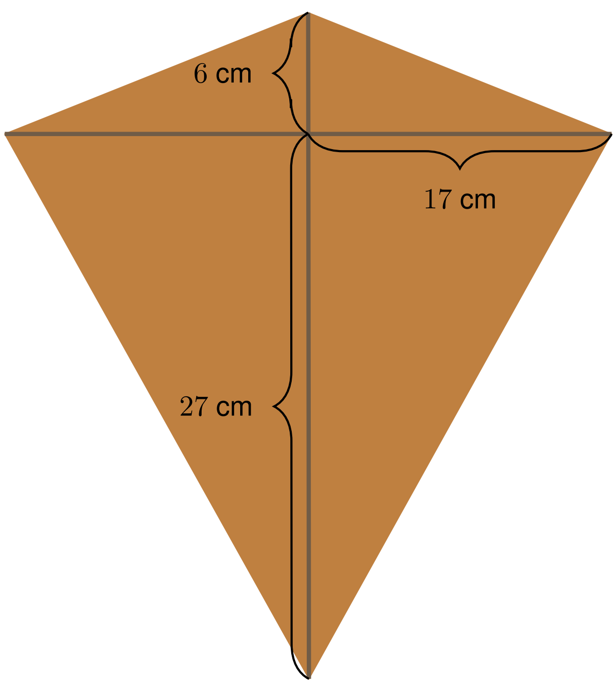
Hoʻolele Lupe Worksheet

1) Create functions which model the following hoʻolele lupe designs. State the domain and range of each function and graph. [Hint: Center the shapes at the origin]:

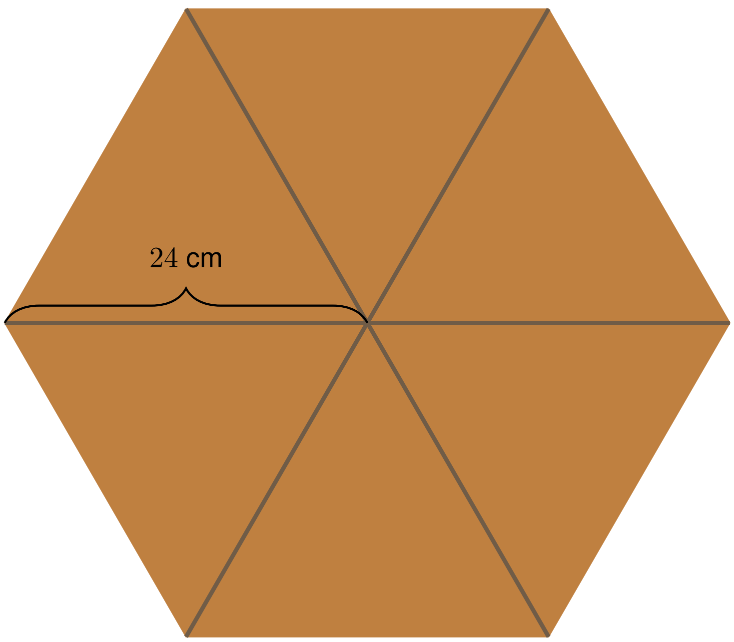
   (a) Diamond/rhombus hoʻolele lupe:



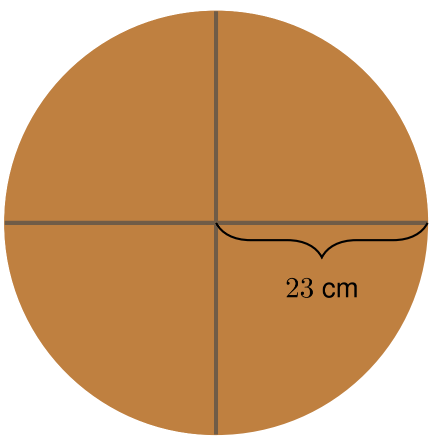
   (b) Kite shaped hoʻolele lupe:



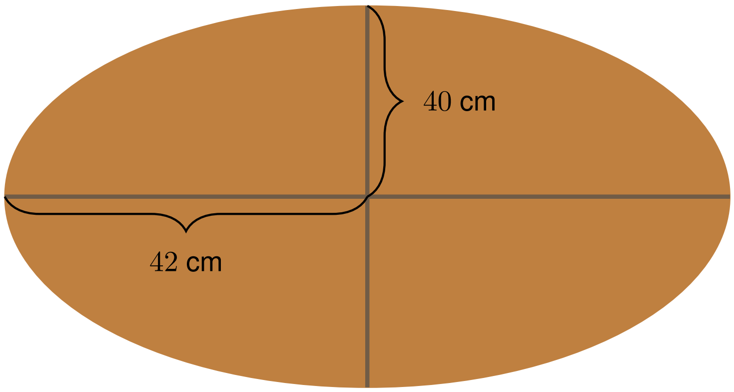
(c) Hexagonal hoʻolele lupe:



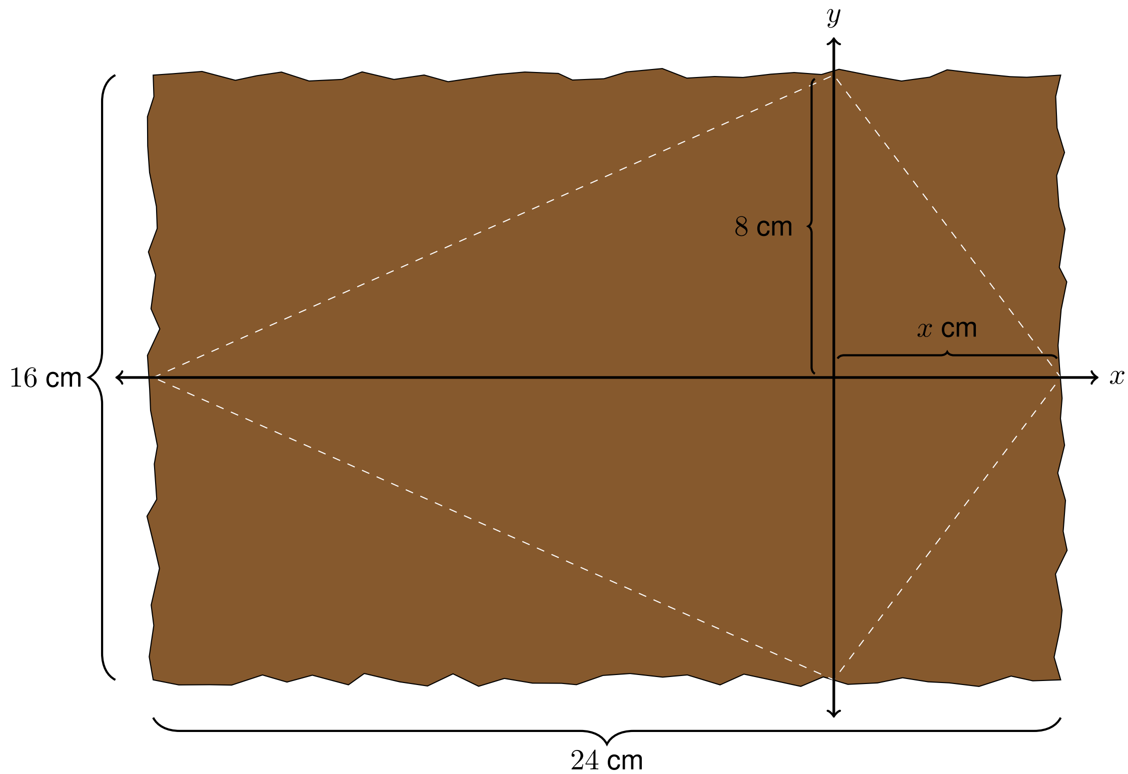
(d) Circular hoʻolele lupe:



(e) Elliptical ho‘olele lupe:



2) Using a rectangular piece of *kapa* to create a kite shapped *ho'olele lupe*, we want to figure out where to cut in order to obtain the biggest piece we can. Use the graphic to answer the following questions.



(a) Create a function $A(x)$ that gives you the area of the kite shapped ho'olele lupe cut from the rectangular kapa with respect to x as defined in the graphic.

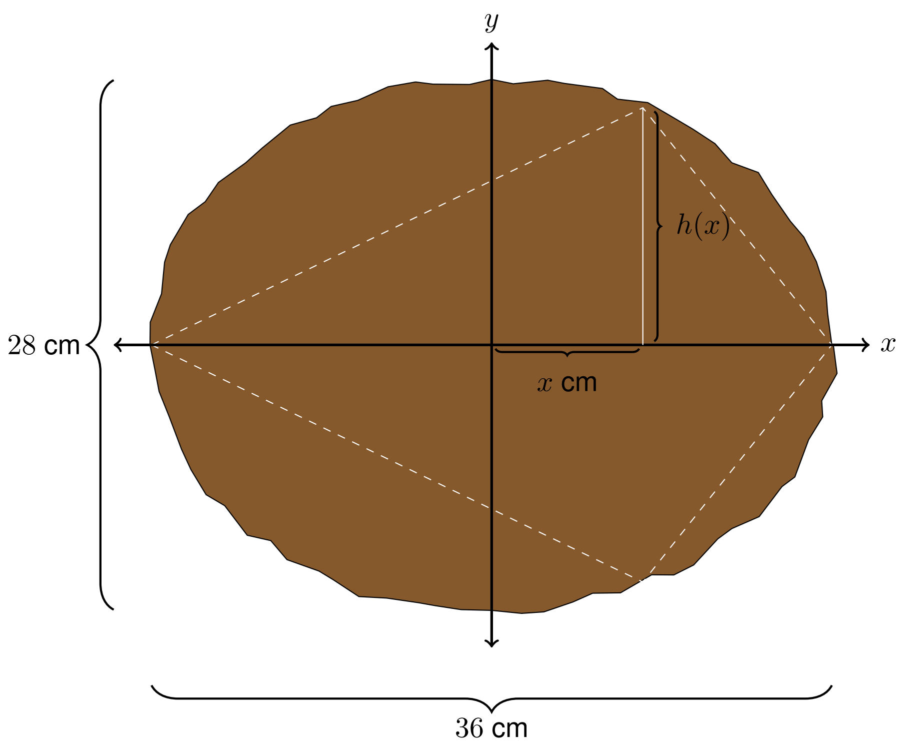
(b) What type of function is it (i.e. Constant, linear, quadratic, etc...)?

(c) State the domain and range of $A(x)$.

(d) Graph $A(x)$.

(e) Use the graph to determine what the maximum area the kite can be with respect to x.

3) Now we have an elliptical shaped of *kapa* 36cm wide and 28cm high and are trying to maximize the area of the kite shaped ho‘olele lupe we can make.



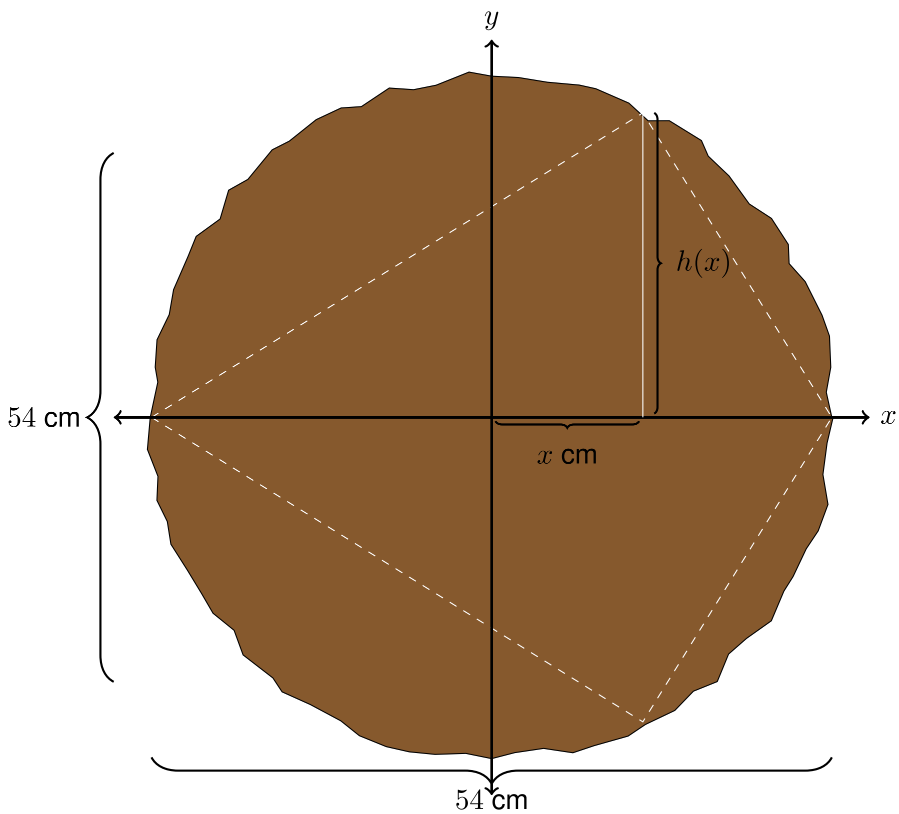
(a) Create a function $A(x)$ that gives you the area of the kite cut from the elliptical kapa with respect to x as defined in the graphic. (Hint: Determine $h(x)$)

(b) State the domain and range of $A(x)$.

(c) Graph $A(x)$.

(d) Use the graph to determine what the maximum area of our kite can be with respect to x.

4) Similar to the last problem, this time the kapa is a circular shape with. Use the diagram to:



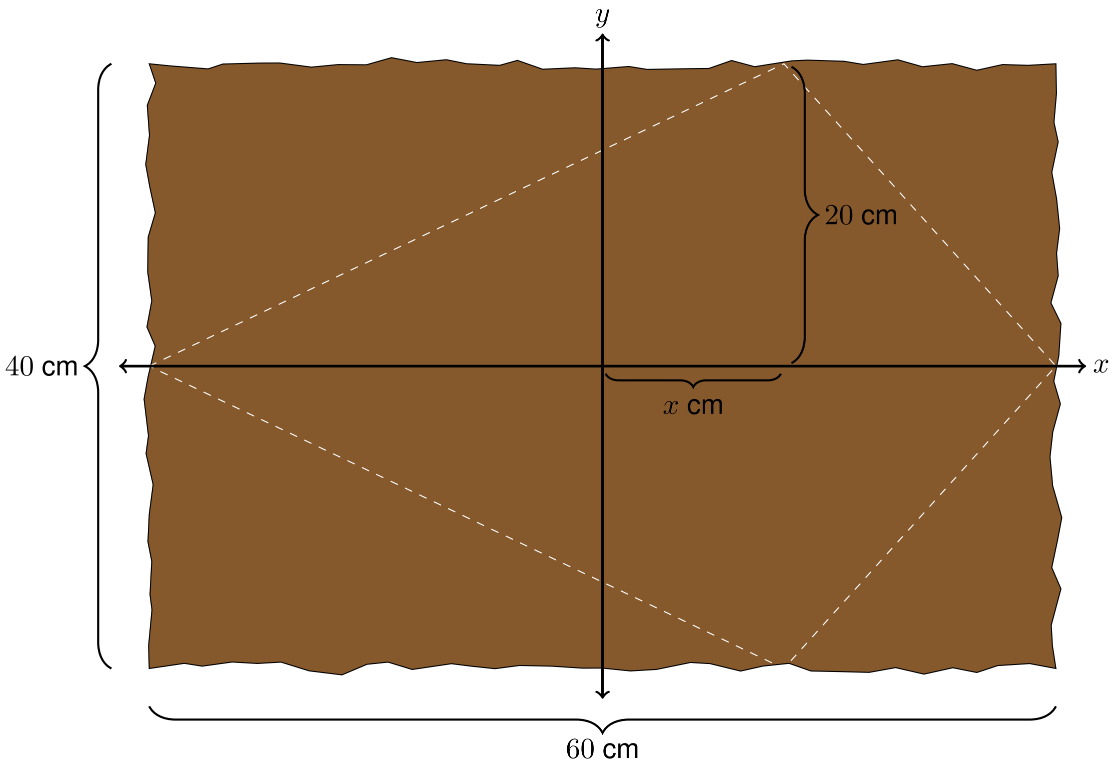
(a) Create a function $A(x)$ that gives you the area of the kite shapped ho'olele lupe cut from the circular kapa with respect to x as defined in the graphic. (Hint: Determine $h(x)$)

(b) State the domain and range of $A(x)$.

(c) Graph $A(x)$.

(d) Use the graph to determine what the maximum area of our kite can be with respect to x.

5) Similar to #1 however this time create a formula that gives you the perimeter of the ho‘olele lupe $P(x)$ to answer the following questions:



(a) What is $P(x)$?

(b) What is the domain and range of $P(x)$?

(c) What is $P(10)$?

(d) Figure out x that gives a perimeter of 150cm.

(e) Determine the max and min of $P(x)$ on the domain you stated in part b. [Hint: you may want to use a graphing utility to graph it first].

1 Answer Key

1. (a) $f_1(x) = -|\frac{21}{20}x| + 21, f_2(x) = |\frac{21}{20}x| - 21$

   (b) $f_1(x) = -|\frac{6}{17}x| + 6, f_2(x) = |\frac{27}{17}x| - 27$

   (c) $f_1(x) = \begin{cases} \sqrt{3}x + 24\sqrt{3}, & \text{if } -24 \leq x < -12 \\ 12\sqrt{3}, & \text{if } -12 \leq x \leq 12 \\ -\sqrt{3}x + 24\sqrt{3}, & \text{if } 12 < x \leq 24 \end{cases}, f_2(x) = \begin{cases} -\sqrt{3}x - 24\sqrt{3}, & \text{if } -24 \leq x < -12 \\ -12\sqrt{3}, & \text{if } -12 \leq x \leq 12 \\ \sqrt{3}x - 24\sqrt{3}, & \text{if } 12 < x \leq 24 \end{cases}$

   (d) $f_1(x) = \sqrt{529 - x^2}, f_2(x) = -\sqrt{529 - x^2}$

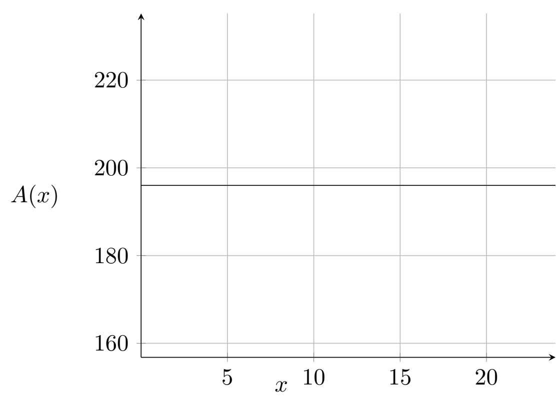
   (e) $f_1(x) = \frac{20}{21}\sqrt{1764 - x^2}, f_2(x) = -\frac{20}{21}\sqrt{1764 - x^2}$

2. (a) $A(x) = 2(\frac{1}{2}(8)(x)) + 2(\frac{1}{2}(8)(24 - x)) = 196$

   (b) Constant Function

   (c) Domain: $(0, 24)$; Range: $\{196\}$

   (d)



   (e) 196cm^2

3. (a) $A(x) = 2((\frac{1}{2})(36)(h(x))) = 252\sqrt{4 - \frac{x^2}{81}}$

   (b) Domain: $(-18, 18)$; Range: $(0, 504]$

   (c)

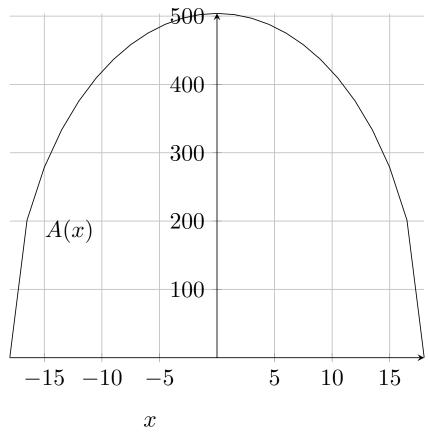
   (d) 504cm^2

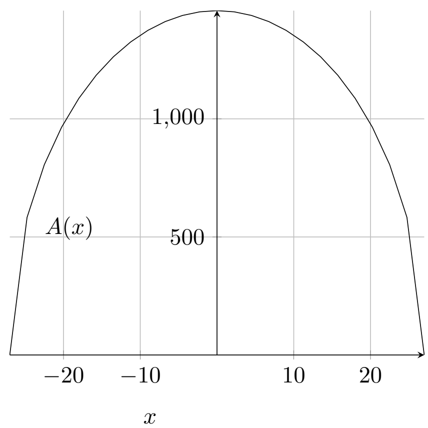
4. (a) $A(x) = 2((\frac{1}{2})(54)\sqrt{729 - x^2} = 54\sqrt{729 - x^2}$

   (b) Domain: $(-27, 27)$; Range: $(0, 1458]$

   (c)

   (d) 1458cm^2





5. (a) $P(x) = 2\sqrt{400 + (x + 30)^2} + 2\sqrt{400 + (x - 30)^2}$
   (b) Domain: $(-30, 30)$
   (c) $40(\sqrt{2} + \sqrt{5}) \approx 146.011$
   (d) $x = \{-\frac{25\sqrt{17}}{6}, \frac{25\sqrt{17}}{6}\}$
   (e) Max of $P(x)$ is $40 + 40\sqrt{10} \approx 166.491$ at $x = \{-30, 30\}$
   and min of $P(x)$ is $40\sqrt{13} \approx 144.222$ at $x = 0$