Hoʻolele Lupe

Introduction

The Hawaiian demigod Maui is given credit for the invention of the hoʻolele lupe (kite). Oral traditions, as well as the creation chants of Kaelikuahulu, suggest that hoʻolele lupe, made of hau covered with kapa or pandanus leaf, were an important part of Native Hawaiian society. Not only were they flown for recreational purposes by young and old men, but they were used for maritime propulsion, fishing, and fighting as well. Hoʻolele lupe might also have held religious significance, as evidenced by the effort the missionaries used to suppress its use (Hoʻolele Lupe-An Analysis of the Ancient Practice of Hawaiian Kite-flying, Damion Sailors).

Grade Levels and Topics

This worksheet is intended for high school students studying calculus. The main topics include critical points, finding global and local maxima, and integration.

- Calculus:
  - **Determining critical points:** $f'(x) = 0$
  - **Differentiation**
  - **Using critical points to determine extreme and local maxima:** If $f'(a) = 0$ and $f''(x) < 0$ then $f(a)$ is a local maxima (possibly extreme maxima).
  - **Integration**

Objectives

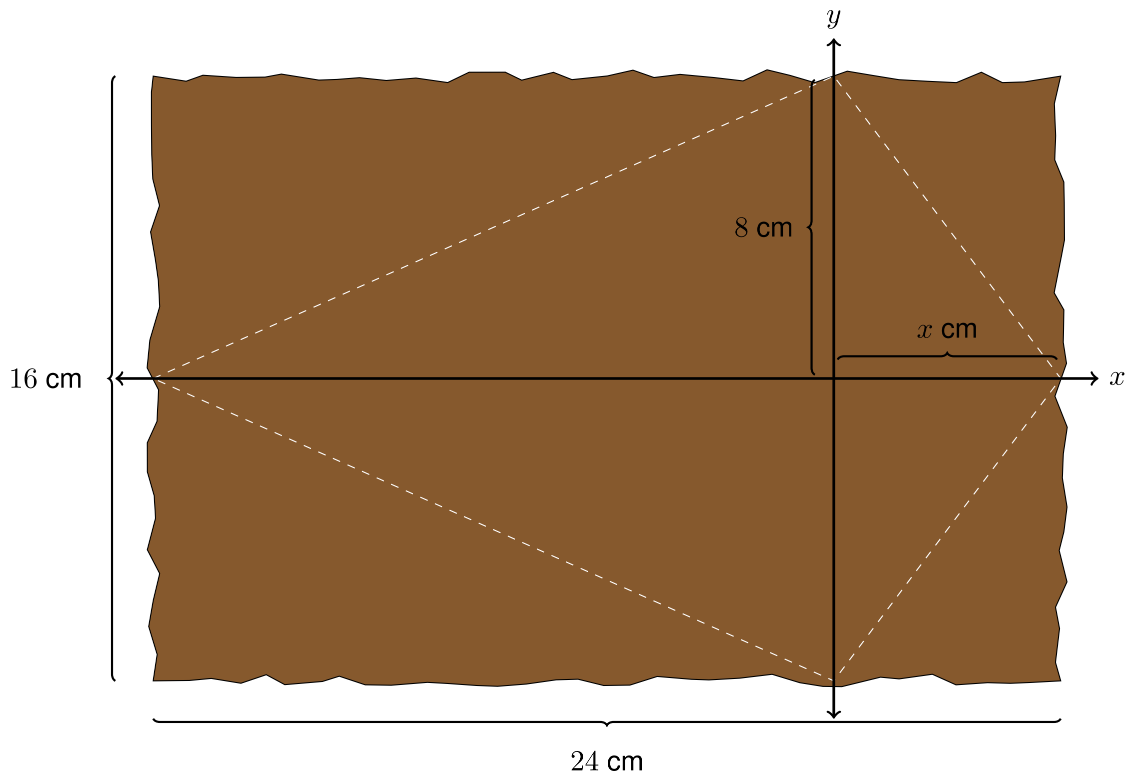
Students will derive formulas for areas that they have not come across before while creating functions to help them optimize materials. In addition, they will apply calculus concepts to tangible questions.

Extensions

To make this activity more interactive, students can also be given cloth sized at the dimensions in the worksheet problems. With only one piece of cloth, they must cut out a kite (of the desired shape) with the maximum amount of area.

Hoʻolele Lupe Worksheet **Name:** ______________________________

1) Using a rectangular piece of *kapa* to create a kite shapped *hoʻolele lupe*, we want to figure out where to cut in order to obtain the biggest piece we can. Use the graphic to answer the following questions.



(a) Create a function $A(x)$ that gives you the area of the kite shapped hoʻolele lupe cut from the rectangular kapa with respect to x as defined in the graphic.

(b) State the domain and range of $A(x)$.

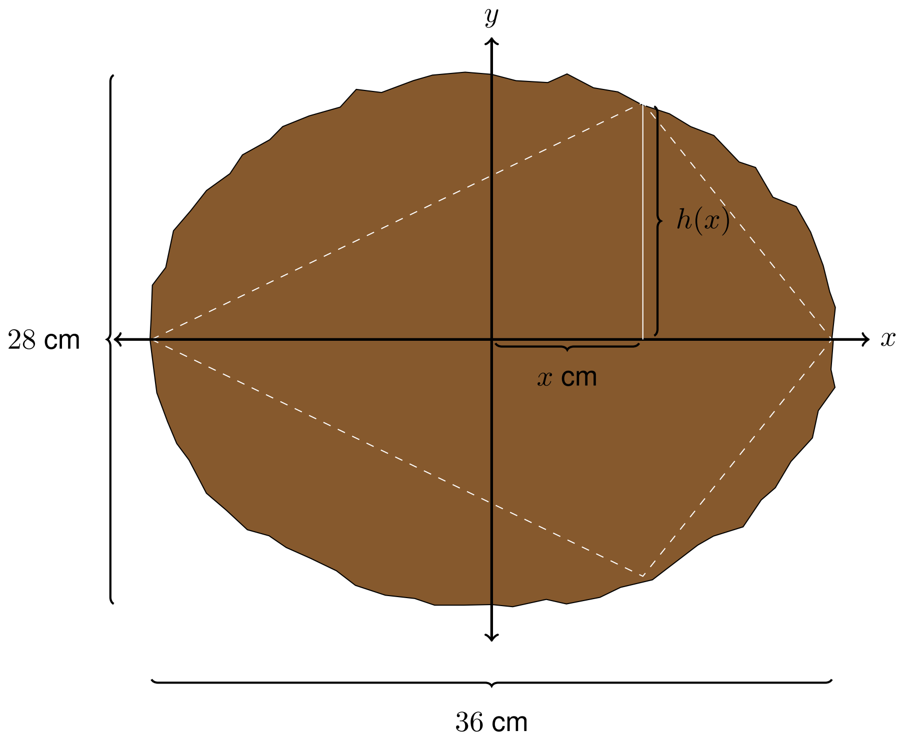
(c) What are the critical points?

(d) Are any of these critical points inflection points? If so, which?

(e) Use the critical points to determine the maximum of $A(x)$.

(f) Graph $A(x)$.

2) Now we have an elliptical shaped of *kapa* 36cm wide and 28cm high and are trying to maximize the area of the kite shaped hoʻolele lupe we can make. [Hint: Use the following image as a guide and find $f(x)$ first]



(a) Create a function $A(x)$ that gives you the area of the kite cut from the elliptical kapa with respect to x as defined in the graphic. (Hint: Determine $h(x)$)

(b) State the domain and range of $A(x)$.

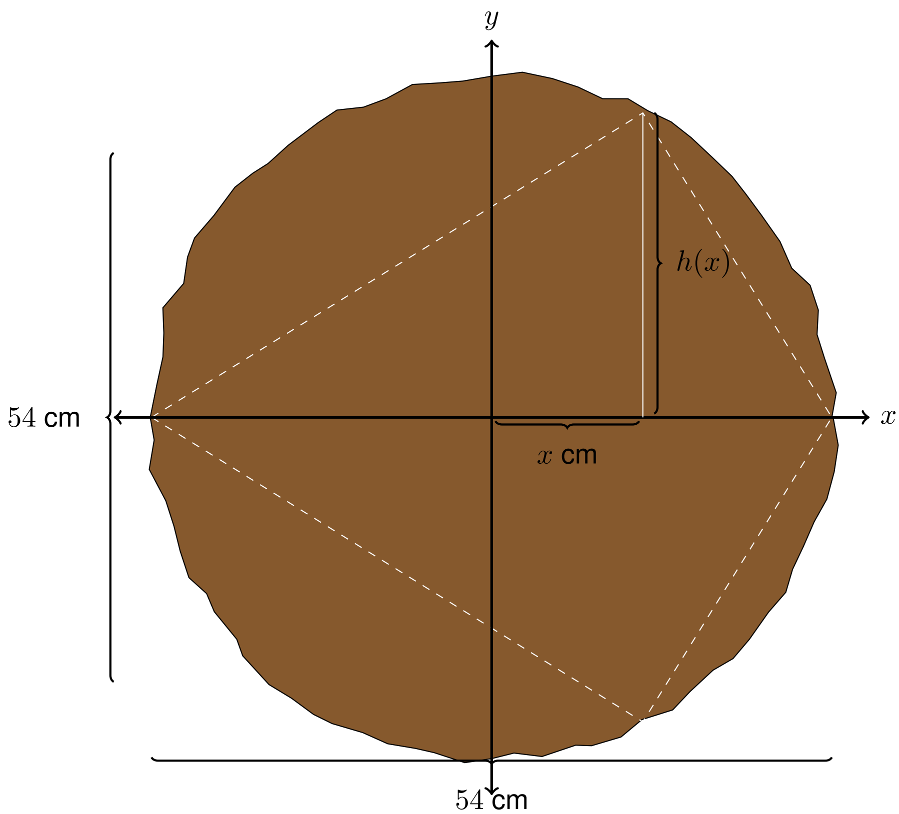
(c) What are the critical points?

(d) Are any of these critical points inflection points? If so, which?

(e) Use the critical points to determine the maximum of $A(x)$.

(f) Graph $A(x)$.

3) Similar to the last problem, this time the kapa is a circular shape with. Use the diagram to:



(a) Create a function $A(x)$ that gives you the area of the kite shapped ho'olele lupe cut from the circular kapa with respect to x as defined in the graphic. (Hint: Determine $h(x)$)

(b) State the domain and range of $A(x)$.

(c) What are the critical points?

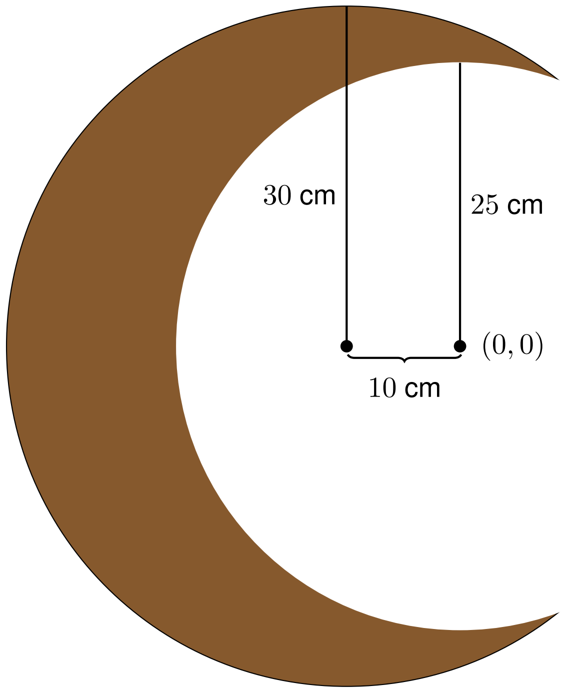
(d) Are any of these critical points inflection points? If so, which?

(e) Use the critical points to determine the maximum of $A(x)$.

(f) Graph $A(x)$.

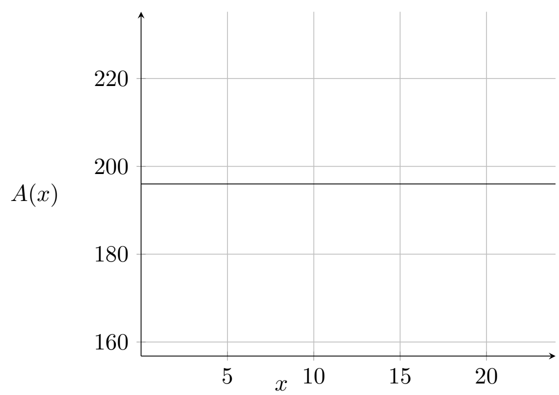
Challenge:

4) Use definite integrals to determine the area of the following crescent shaped hoʻolele lupe.

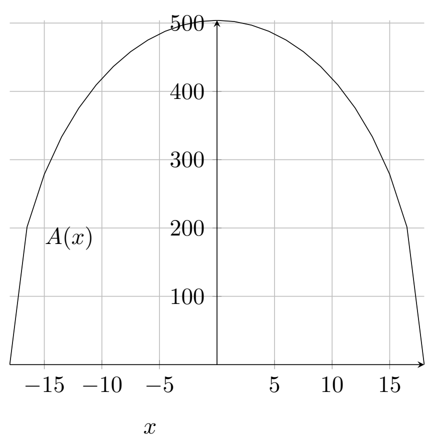


Answer Key

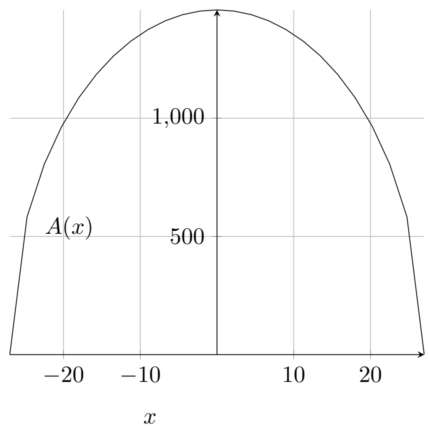
1. (a) $A(x) = 2(\frac{1}{2}(8)(x)) + 2(\frac{1}{2}(8)(24-x)) = 196$
   (b) Domain: $(0, 24)$; Range: $\{196\}$
   (c) Critical point at $A(0) = 196$
   (d) No.
   (e) 196cm^2
   (f)



2. (a) $A(x) = 2((\frac{1}{2})(36)(h(x))) = 252\sqrt{4 - \frac{x^2}{81}}$
   (b) Domain: $(-18, 18)$; Range: $(0, 504]$
   (c) Critical point at $A(0) = 504$
   (d) No.
   (e) 504cm^2
   (f)



3. (a) $A(x) = 2((\frac{1}{2})(54)\sqrt{729 - x^2} = 54\sqrt{729 - x^2}$
   (b) Domain: $(-27, 27)$; Range: $(0, 1458]$
   (c) Critical Point at $x = 0$
   (d) No.
   (e) Max of 1458 at $x = 0$
   (f)



4. $2\left(\int_{-30}^{\frac{75}{4}} \sqrt{900 - x^2}dx - \int_{-15}^{\frac{75}{4}} \sqrt{625 - (x - 10)^2}dx\right)$
$= 2\left(\left(\frac{x}{2}\sqrt{900 - x^2} + 450\sin^{-1}\left(\frac{x}{30}\right)\right)\Big|_{x=-30}^{\frac{75}{4}} - \frac{1}{2}\left((x - 10)\sqrt{625 - (x - 10)^2} - 625\sin^{-1}\left(\frac{2}{5} - \frac{x}{25}\right)\right)\Big|_{x=-15}^{\frac{75}{4}}\right)$
$\approx 2\,(1230.2182 - 705.0718) = 1050.2928$cm^2