Infection Model Simulation

Grade Levels

This activity is intended for grades $6-12$.

Objectives and Topics

This purpose of this lesson is to develop a model and simulation of an epidemic, to apply graphical and statistical analysis to the results, to analyze the merit of approximated models. Math topics involved in this lesson include: modeling, graphing, statistics, biology.

Activity Length

This lesson will take $2-3$ periods, depending on depth.

You may find the notes on stage models useful as a lead-in to this activity, however, it is not necessary. We will simulate the spread of a disease through a migrating population.

Materials

- Plastic Easter Eggs (to represent people)
- Egg cartons (to represent planes)
- Colored circle stickers (to mark infection)

You will need several easter eggs for each group (at least 16 per groups). *After easter, there are plenty of eggs on sale!.* Egg cartons should be trimmed with scissors to remove the cover and vary the number of eggs that can be held in each (I use planes that are 3×3, 2×3, and 2×4).

Alternatives to easter eggs: Any typical counter will work. Even slips of paper rolled into cylinders or post-it notes would suffice.

The Simulation

1. Split students into several groups ($2-3$ students per group) and spread them around the room. Distribute the cartons, eggs, and stickers to each group. Each group will represent an airport hub.

2. Begin the epidemic with a single infected individual (marked with a sticker on the egg) in one of the planes.
3. Have groups trade planes randomly for a few seconds.
4. After trading, any uninfected egg sitting directly adjacent (forward, back, left, right) of an infected individual should be marked as infected with a sticker.
5. Have each group shuffle the position of the eggs in their planes (only within the group – not with other groups). This way, when the planes fly again, the infection will spread wider.
6. Have each group record the number of infected passengers they have at their hub.
7. Repeat steps $3 - 6$ several times until the infection has spread to all individuals.
8. Record data from each hub on a table.
9. Graph infection numbers (either total or by hub).
10. Graph the rate of infection (# of new infected per trade)

Questions

- Did the infection spread as you would have expected? Was it faster/slower?
- How would you make the simulation more realistic?
- How would you control the outbreak?
- The model contains an element of randomness in the shuffling of eggs between planes. If we removed this randomness and carefully chose where to seat people, is it possible to contain the infection? Is it possible for a single individual to always avoid infection?
- Why are mathematical models and simulations important in real life?

Some Notes

- The infection will start slowly. As the infected numbers increase, the rate of infection will also increase up until about half of everyone is infected. At this point, the rate of infection will decrease (because the chance of infected people sitting next to uninfected people will decrease). Ideally, the infection numbers for the full population will follow a logistic curve.

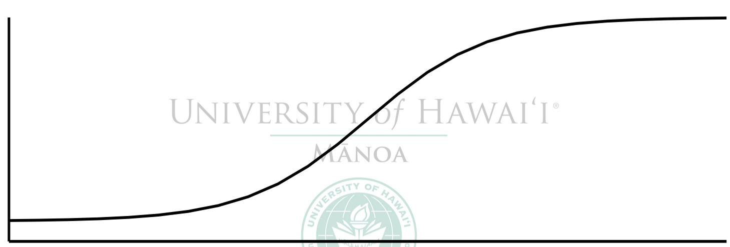

Figure 1: A logistic curve. Infection should ideally progress something like this.

- Despite the fact that it's not going to be linear, ask the students to linearize the model data (find a line of best fit). What does the slope mean? *(average rate of infection.)* What does the y-intercept mean? *(initial infected population.)* Why is a linear model insufficient for this case?
- The trading of planes and shuffling of passengers is random, so the model is not deterministic. You can run the simulation several times and compare infection numbers, infection rates, and graphs (calculate means, medians, modes).
- It is unrealistic that people continually fly from airport hub to airport hub. Uninfected passengers aren't an issue: an uninfected passenger gets off the plane and is replaced by a new uninfected passenger – this doesn't change anything in the model. The infected passengers are more of an issue since the chance that an infected passenger is replaced by an infected passenger is relatively small. However, an infected individual will go on to infect their community, making the chances that an infected person will board a plane *eventually* much higher.
- Don't worry about the equations behind this simulation. The random trading and shuffling means this is *not* a straightforward stage-model. You may contact the author aaronts@hawaii.edu if you are interested in learning how to develop the equations that approximate this simulation.
- To respond to the last question above: Mathematical models are important tools in planning a response to events which cannot be empirically tested, particularly where biology is concerned. For ethical reasons, you cannot start a real epidemic just to study how it behaves. Nor can we simply wait around for an epidemic to break out and simply react on the fly. To efficiently save as many lives as possible (or in the case of non-lethal diseases, to minimize the economic impact), we must have a plan ahead of time, and models and simulations allow us to test what the best response is.

Complicating the Model

In response to the question "How would you make the model more realistic?", the simulation can be altered. Here are a few examples.

Airline Network

Not every airport has flights to every other airport. Try setting specific flight paths for your simulation (ie. each group can only trade planes with certain other groups). This may affect infection rates. Perhaps relatively isolated airports will will stay cleaner longer, while busy hubs are swimming in disease.

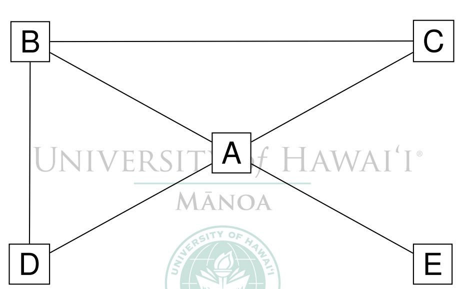

Figure 2: An example airline network. Hub A is connected to everyone. Hub E is more isolated.

Virulence

Realistically, infection does not have a 100% chance of spreading to a neighbor. Try flipping a coin or rolling a die and only infecting a neighbor with some probability. This should slow infection. Alternatively (or additionally), give a chance to infect not just immediate neighbors but eggs two spaces away, or even the entire plane.

Recovery

People eventually get better. If you mark each sticker you apply with the time of infection, you can simulate recovery by removing stickers that are old enough. In other words, if an egg is infected in round 3, apply a sticker with a 3 on it. Then, when you get to round 6 (for example), remove all stickers with a 3 on it (of course, all stickers with a 2 or 1 written on them will have already been removed in the previous rounds). Different lengths of infection will affect how explosive the outbreak is.

Control

As the SIR model suggests, there are two general ways to contain epidemics: remove susceptible individuals or remove infected ones. Generally, quarantine removes infected ones, vaccination removes susceptible individuals. Keep in mind that the aim is to control infection, not eliminate it. Eradication of a disease is extremely difficult in real life; usually, we’re satisfied to merely keep numbers low.

Quarantine

Institute a quarantine at a hub by removing any infected who arrive (simply remove the sticker – assume infected passengers are replaced by uninfected passengers). If you do this at every hub with 100% reliability, then the infection will stop immediately. Realistically, however, individuals get through. Maybe institute a quarantine at only a few hubs or flip a coin or roll a dice for removal.

Vaccination

Institute a vaccination program by marking uninfected eggs as vaccinated (use a specific sticker color/type). These eggs are now immune to infection. Again, if every egg is marked as vaccinated, then the infection will stop immediately, so institute it at only certain hubs OR give only a limited supply of vaccine (stickers).