Invasive Species

Grade Levels

This activity is intended for grades $9 - 12$.

Introduction

Located some $2,400$ miles from the nearest continental shore, the Hawaiian Islands are the most isolated group of islands on the planet. The plant and animal life of the Hawaiian archipelago is the result of early, very infrequent colonizations of arriving species and the slow evolution of those species (in isolation from the rest of the world's flora and fauna) over a period of at least 70 million years. As a consequence, Hawai‘i is home to a large number of endemic species. The restricted scale, isolation, and sharp boundaries of islands create unique selective pressures. The reduced diversity of species on islands results in a reorganization of species interactions within island communities, and island species are generally subjected to reduced predation and competition between species. Unfortunately, due to the lack of predation, islands species evolve and lose their defense mechanisms, making them very susceptible to invasive species. Because of the relatively small area involved (islands versus a continent, for example), many Hawaiian species are considered threatened even when at their normal population levels.

Hawai‘i has a growing invasive species crisis affecting the islands' endangered plants and animals, overall environmental and human health, and the viability of its tourism and agriculture-based economy. Invasive species occur globally, but Hawai‘i is more susceptible to invasive species because it is an island. The entire island chain of Hawai‘i has been devastated by invasive insects, plants, hoofed animals such as deer, goats and pigs and others. Invasive species threaten biodiversity by causing disease, acting as predators, parasites, or competitors, altering habitat, and/or hybridizing with local species. Invasive predators often move to the top of the food chain and disrupt prey populations, particularly for small mammals, birds, insects, and plants. From an economic perspective, the cost to control invasive species and the damages they inflict upon property and natural resources in the U.S. is estimated at $\$137$ billion annually. However, one cannot put a price on the permanent loss of native animals and habitat in Hawai‘i. Many endemic creatures have become extinct within the last century. For example, there are 71 known taxa of endemic Hawaiian birds, of which 23 are extinct and 30 of the remaining 48 species and subspecies are listed as endangered or threatened.

Problems

1. Coqui frogs were accidentally introduced into Hawai'i from Puerto Rico in about 1988. Aside from being a major noise nuisance, the frogs pose a threat to the island's ecosystem. Coqui frogs have a voracious appetite that puts the island's unique insects and spiders at risk. They also compete with endemic birds and other native fauna that rely on insects for food. The frogs are also quite adaptable to the different ecological zones and elevations in the state and have been found from sea level to $4,000$ feet elevation.

   Now, a single female frog can produce more than $1,400$ eggs per year. Assume the island is a closed environment, and that the frogs have no natural predators.

   (a) Describe the population of coqui frogs on the island of Hawai'i over time.

   (b) As recently as May 2010, coqui frogs had been heard in Mānoa Valley on O'ahu, inciting a major frog hunt for state wildlife officials (successfully). Suppose that an adult male and female were introduced. Fill out the table for the first 4 years. Assume that half of all new frogs born every year are female and that all frogs survive birth.

Time	Number of Females	Number of Males	Total Population
0	1	1	2
1			
2			
3			
4			
$\vdots$	$\vdots$	$\vdots$	

   (c) Based on the data above, the population of coqui frogs in any year has been determined to be modeled by a polynomial generated by a Pascal's Triangle with coefficient multiplied by 2, and then evaluated at 700:

Table 1: Modified Pascal's Triangle

$t = 0$:					2				
$t = 1$:				2		2			
$t = 2$:			2		4		2		
$t = 3$:		2		6		6		2	
$t = 4$:	2		8		12		8		2

   For example, the polynomial for year one would be $2x + 2$, giving a population size of

$$P_1 = 2(700) + 2 = 1402.$$

   Using this information, determine the population of coqui frogs in year five.

2. Laysan Island is located 790 sea miles to the northwest of Honolulu in the Papahānaumokuākea Marine National Monument. The island is shaped like a large Hawaiian poi-pounding board or oval serving dish, about a mile wide by two miles long, north and south. The island was found to have huge guano deposits which were mined. To supply the miners with food, manager Max Schlemmer introduced rabbits in 1903. Assume the island is a closed environment, and that the rabbits have no natural predators.

   (a) Describe the population of rabbits over time on Laysan Island.

   (b) Suppose that an adult male and female were introduced to Laysan in 1903. Keep in mind that rabbits can reproduce every 30 days and that the average size of the litter is about 8 babies (half of which are female).
   Fill out the table for the first 6 months. Use the Pascal Triangle method from before.

Time	Number of Females	Number of Males	Total Population
0	1	1	2
1			
2			
3			
4			
5			
6			

   (c) Does the data in the table in (b) match the description in part (a)? Why or why not?

3. How is the situation with coqui frogs different from the rabbits? Describe in detail.

4. Laysan Island was preserved as a bird sanctuary in 1909. An expedition to Laysan Island in 1911, a mere 8 years after the introduction of rabbits, found that the rabbits had killed many bushes and nearly exterminated several plant species. The rabbits were likely to eat out the vegetation, which would result in the disappearance of the insects on which a number of endemic bird species were dependent, thus they needed to be eradicated. After killing $5,000$ of them, the hunters realized the difficulty of finding and killing all of them and left. In 1923, the Tanager Expedition visited Laysan; the island by then had been reduced to a barren waste of sand with a few stunted trees, only 4 of the 26 species of plants recorded from the island were found; and there were a few hundred rabbits present. These were shot, the last ones being hunted out individually. Unfortunately, the damage had been done. The endemic warbler had vanished; the last three Laysan honeyeaters died during a sand storm while the expedition was on the island; and the Laysan rail died out shortly afterwards. Extermination of the rabbits was completed and no sign of them was seen upon return to the island in 1936, when the island was found to be recovered in vegetation.

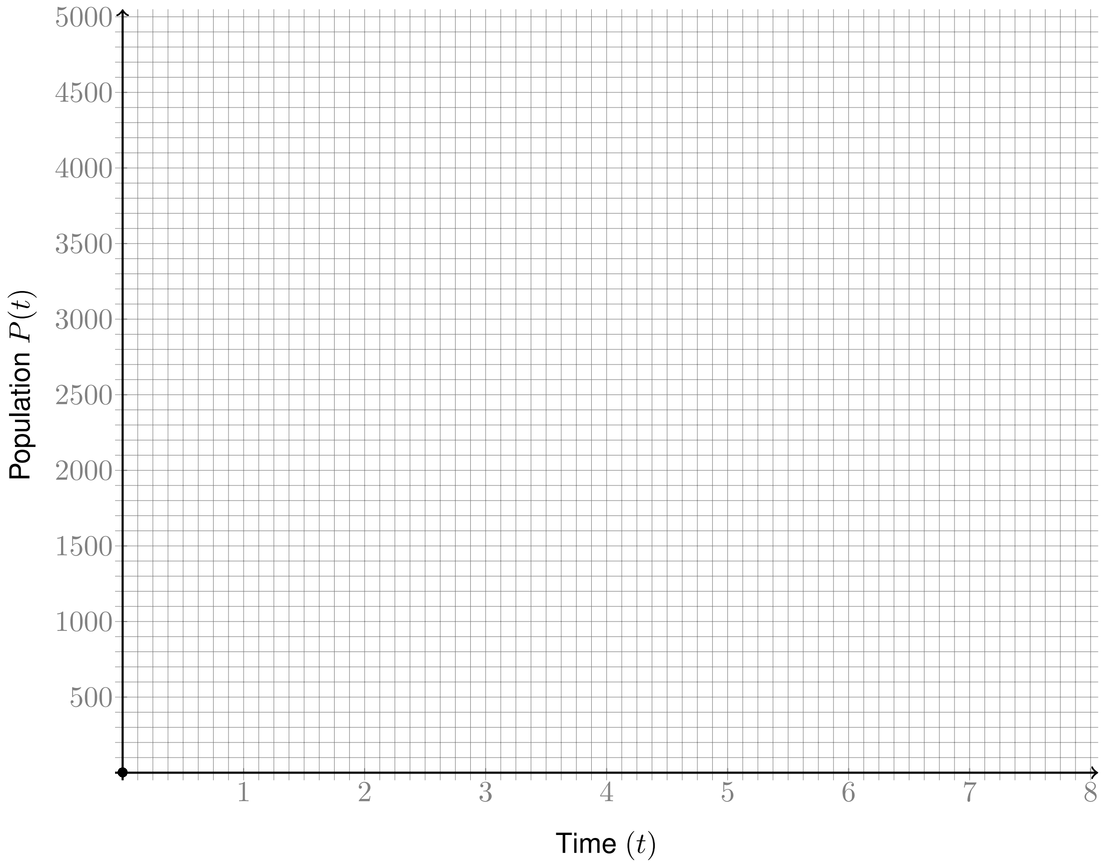
   In a closed environment, like Laysan Island, with adequate but not unlimited resources and no predators, a population of rabbits can be modeled by

$$P(t) = \frac{K}{1 + (\frac{K}{P_0} - 1)e^{-rt}},$$

   where P_0 is the initial population, K is the carrying capacity, and r is the rate at which the population grows. Suppose that $P_0 = 2$, $K = 5,000$, and $r = 2$.

(a) Write the equation that models the population as a function of time.

(b) Plot this function. The initial point has already been plotted.



(c) By looking at the graph, during what year does:

i. $P \geq 100$

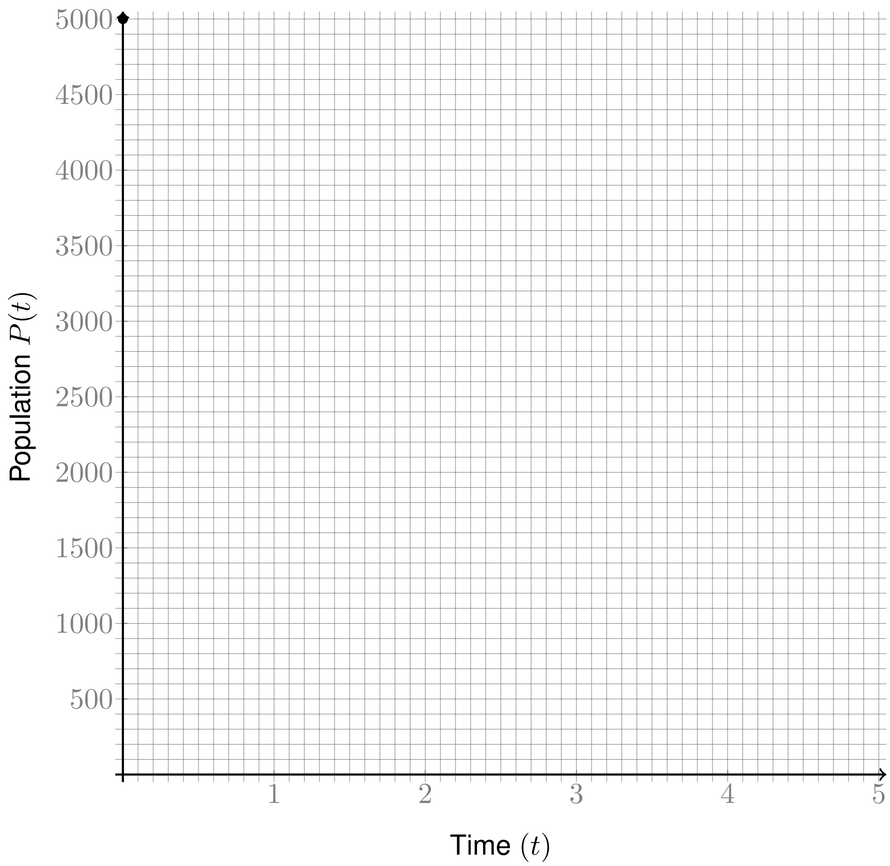
ii. $P \geq 500$

iii. $P \geq 3,000$

(d) Describe the long-term trend of the population.

(e) Why do we call K the "carrying capacity"? What do we mean by this term?

(f) Obviously, due to the rapid growth of the rabbit population, Laysan Island would be completely denuded of plants, and the rabbits would begin to starve at a rapid rate. Assume that the death of the rabbits can be modeled by an exponential equation of the form $P(t) = P_0 e^{rt}$, where P_0 is the initial population and r is the rate at which the population declines. Suppose $P_0 = 5,000$ and $r = -250\% = -2.5$. Graph this function.



How long does it take for:

i. $P \leq 1,000$

ii. $P \leq 100$

iii. $P \leq 1$ (This is when the rabbits are officially eliminated.)

Solutions

1a With no predators and essentially limitless space and food, the population will grow exponentially. The only limiting factor to the population size is naturally-occurring death and disease.

1b Table:

Time	Number of Females	Number of Males	Total Population
0	1	1	2
1	701	701	1402
2	$491,401$	$491,401$	$982,802$
3	3.44×10^8	3.44×10^8	6.88×10^8
4	2.41×10^{11}	2.41×10^{11}	4.82×10^{11}
$\vdots$	$\vdots$	$\vdots$	

1c The $t = 5$ row of the triangle would be 2102020102, yielding a polynomial $2x^5+10x^4+20x^3+20x^2+10x+2$. Evaluated at $x = 700$ gives a population size of 3.99×10^{14}. That is 399 ***trillion***!!!

2a Just like the coqui frogs, the rabbits will reproduce unchecked. However, unlike the situation before, on such a small island, there is limited space and resources. The population will reach a maximum size that the island resources will support. When all resources are depleted, the population will fall to a lower level until the plants can grow back. The population will grow to a maximum, but much lower than the previous maximum. The population will again decrease. This process will repeat many times until either all of one of the sexes will die off, or the population reaches a manageable size that allows time for the plants to grow back fast enough to support the population.

2b Table:

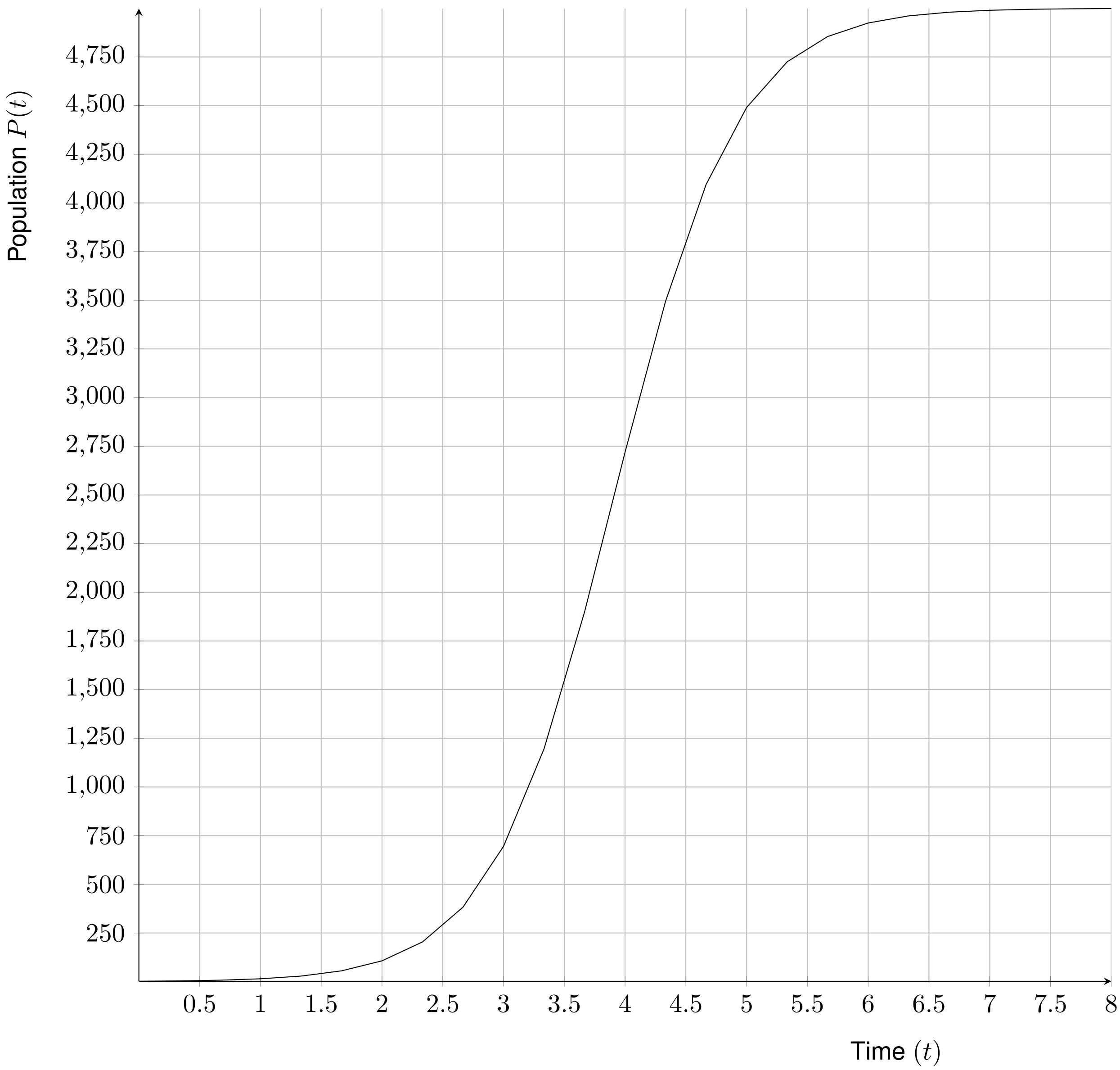
Time	Number of Females	Number of Males	Total Population
0	1	1	2
1	5	5	10
2	25	25	50
3	125	125	250
4	625	625	$1,250$
5	$3,125$	$3,125$	$6,250$
6	$15,625$	$15,625$	$31,250$

2c The data does not match the intuition. The table does not take into account rabbit death, access to resources, or availability of space.

3 See part (a) of Questions 1 & 2 for detailed answer.

4a $P(t) = \frac{5{,}000}{1+(\frac{5{,}000}{2}-1)e^{-2t}} = \frac{5{,}000}{1+2{,}499e^{-2t}}$

4b Graph:



4(c) i. When $t \approx 2$.

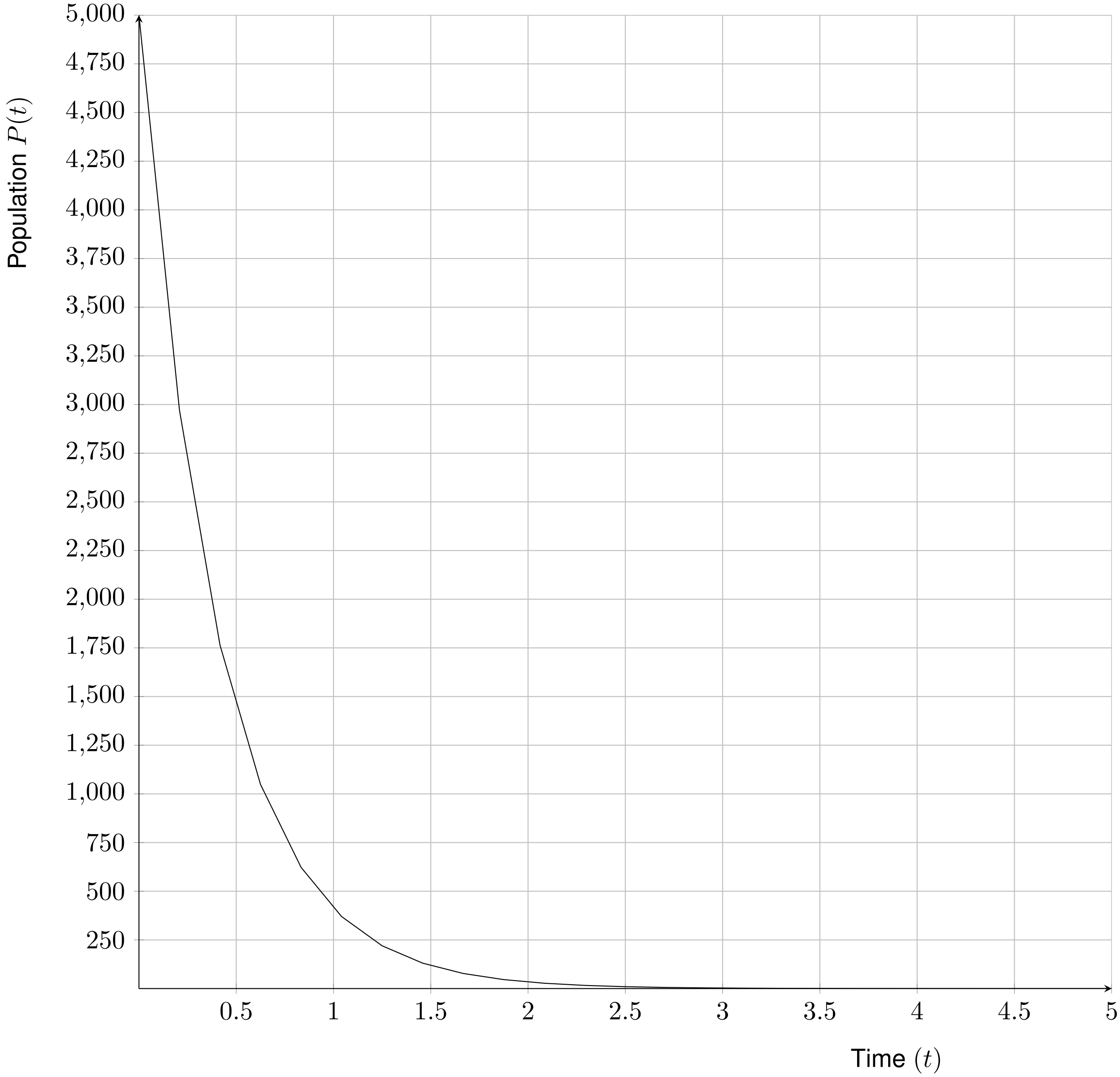
4(c) ii. When $t \approx 2.8$.

4(c) iii. When $t \approx 4.13$.

4c The population will climb to a maximum value, but will not exceed that value. Mathematically, $P \leq 5,000$.

4d It means the largest population that the environment can support.

4e Graph:



4(f) i. When $t \approx 0.6$.

4(f) ii. When $t \approx 1.58$.

4(f) iii. When $t \approx 3.42$.