Logic Puzzles: Functions

Acknowledgement

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Grade Levels

This activity is intended for high school students.

Objectives and Topics

This activity introduces functions to students in a new context other than graphing or mapping real numbers to real numbers. Students will observe an application of functions as well as an informal introduction to mathematical relations.

Materials

- Handouts

Introduction and Outline

Simple logic puzzles like these are a common form of mathematical game. The examples below (see Handouts) demonstrate what are known as **constraint satisfaction problems**. At first these seem like mental exercises that do not incorporate any formal mathematics. However, the solutions to these puzzles demonstrate two fundamental mathematical concepts, functions and relations (we go into detail about the formal mathematical meaning of these terms below). This activity is suitable for any class that has been introduced to the concept of function, in most curricula this will be in the form of functions from real numbers to real numbers. This activity demonstrates that the concept of a function is more versatile than it may at first appear to students.

Before going further, take some time to solve each puzzle in the Handouts section. The first is very simple; the second may take a few tries. The third problem is quite difficult. It will take you much trial and error and a significant amount of time to finish. It is important that you do not allow yourself to be so discouraged that you give up on the problem; this will make it easier to motivate your students. No solutions have been provided. This is to ensure you will understand what the students will be going through as they do the activity.

One can view the solutions you devised to these problems as functions. Suppose you labeled the empty spots in the table for the "three swimmers" problem by row and column starting from the top left (so $1-1$ is the square in the top left, $2-1$ is the left most square in the middle row, $1-2$ is the middle box in top row, etc.). If your solution involved having "1st place" in the top left hand box then your solution is a function that maps the box labeled $1-1$ to the value "1st place". Functions are usually represented by letters, so in our example we will call the function you found for the solution to the three swimmers f. In mathematical shorthand, one writes the statement "the function f maps $1-1$ to 1st place" as

$$f : 1-1 \longrightarrow \text{1st place.}$$

To find a solution to any of the puzzles, one needs to know what values the function takes as inputs (called the domain of the function), the values that these inputs are mapped to (called the range of the function). For the three swimmers, the domain of f is the boxes in the table and the range is the possible placements, swimsuit colors, and names of the swimmers. But the puzzles include relationships on the range of the functions that are solutions. For instance, one of the clues in the three swimmers problem is that the 1st place winner wore a red swimsuit. This means that if a given function is a solution to the problem, then the box that the function maps to "1st place" must be in the same column as the box that the function maps to "red swimsuit." These mathematical relations are the "constraints" in a "constraint satisfaction problem." The easiest way to think about relations like these constraints is as a rule. Continuing with the above example, one can say If the function f is a solution, then f maps boxes from the same column to the values "1st place" and "red swimsuit"

One can view all of the constraints in all three of the problems in a similar manner. Puzzles like these offer students concrete example of constraint satisfaction problems. More complex and abstract constraint satisfaction problems are of great importance in various branches of computer science. Many algebraists spend a great deal of time researching the mathematical properties of the functions and relations involved in different types of constraint satisfaction problems to determine if there are efficient methods of finding their solutions.

Suggestions

Before beginning this activity, it is helpful to do a quick review of the definition of a function and have the students recall examples of functions they are familiar with. Have the students split into small groups and begin working on the problems. The first two problems should be solved in pretty good time, no more than about ten minutes. The third will take a long while and could prove very frustrating, feel free to warn the students about the increased difficulty of this problem before they begin.

After all groups have completed the first two problems and have had some time to consider the third, bring the class together again. Call on students to present their solutions to the first two problems to the entire class. As the students present their solutions, display a labeled version of the boxes they filled in (row by column is as good a labeling as any), and write down their solutions as a series of function statements (i.e. $f : x \longrightarrow y$). Have the students discuss in their groups how their solutions are functions, and what constraints their functions must satisfy in order to be a solution. After some discussion, bring the class together again and have the students share their conclusions.

To finish, summarize how these problems show that the functions need not just be about numbers, but that the concept of a function is a very versatile one. Have the students write their solution to the "four athletes" problem as a function, and rephrase the constraints given in the problem in terms of a function. Have groups share their solutions with the whole class. It is likely the entire class period will be taken up by now, but if you find you have time left over have the students continue to work on the third problem. Otherwise, leave it as a challenge to them, or assign it for homework.

Handouts

Puzzle 1

Three Olympic swimmers took 1st, 2nd, and 3rd in their race at the Olympic Games. When they went to the victory stand each wore a different colored swimsuit. From the clues below tell the name, place and swimsuit color of each of the swimmers.

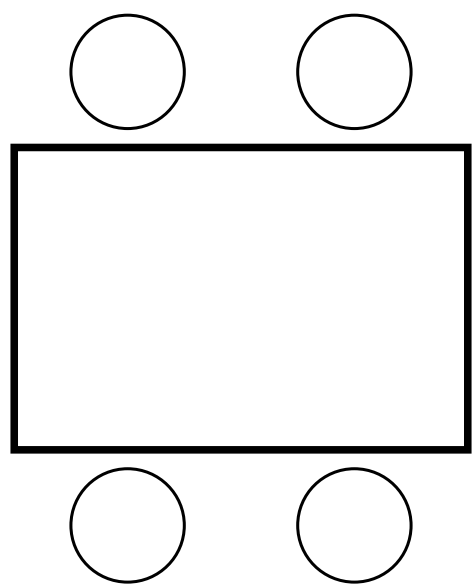
1. The first place swimmer wore a red swimsuit.
2. Tracy took third place.
3. Nancy wore blue.
4. Mary and the girl in the white swimsuit were roommates.

Placement			
Swimsuit			
Name			

Puzzle 2

Four friends are sitting around a table having lunch and discussing their favorite athletic pastimes.
If the following facts are known about the seating arrangement, can you place each person in their position around the table and tell their sport?

1. Bob is sitting across from the tennis player.
2. The golfer is sitting across from Ted.
3. Alice is on Carol’s left.
4. The jogger is sitting on the swimmer’s right.
5. There is a man sitting on Bob’s right.



Puzzle 3

Fill in the grid below with the first nine letters of the alphabet

A, B, C, D, E, F, G, H, I

The following rules apply:

1. No consecutive letters (right next to each other) are in the same row horizontally or the same column vertically.
2. No row or column contains two vowels.
3. B and E are in the same column.
4. F is not in column 3.
5. A is in the bottom row.
6. C and G are in the same row.
7. D is not in the bottom row.
8. I and H are in corner positions
