M&M Madness

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1 Introduction

Exponential and linear functions have drastic differences, but both are very important in understanding the world around us. Exponential functions are the heart of many types of models. E.g. population, radioactive decay, and interest models. Similarly, linear equations are important in other types of models. In particular, any situation where there is a constant rate of change. E.g. cost functions, temperature conversion, and currency exchange. This is why studying these types of functions is important and why knowing the difference between them is equally important.

2 Length and Objectives

The lesson is designed for a 90 minute session. Students will conduct an experiment and make conclusions about how the results relate to known functions.

3 Prerequisites

Students should have a basic understanding of linear and exponential functions. For linear equations, they should be able to take a graph of a line and describe the line as an equation in slope-intercept form. For exponential equations, they should be familiar with the standard form $y = a(b^x)$ and how each variable relates to its graph. In general, students should be able to take experimental data and create a graph that describes the data, as well as a basic knowledge of graphing equations on a calculator.

4 Grade Levels and Topics

This lesson can be applied to any intermediate or high school algebra class. The lesson includes topics in exponential functions, linear functions, experimentation processes, and graphing calculator techniques.

5 Common Core Standards

(Note: This lesson has only been aligned with high school standards.)

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Modeling

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods.

A-CED: Creating Equations

Create equations that describe numbers or relationships.

1. Create equations and inequalities in one variable and use them to solve problems.
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

F-IF: Interpreting Functions

Understand the concept of a function and use function notation.

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

Interpret functions that arise in applications in terms of the context.

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

Analyze functions using different representations.

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. (a. & e.)

F-BF: Building Functions

Build a function that models a relationship between two quantities.

1. Write a function that describes a relationship between two quantities. (a.)

F-LE: Linear, Quadratic, and Exponential Models

Construct and compare linear, quadratic, and exponential models and solve problems.

1. Distinguish between situations that can be modeled with linear functions and with exponential functions. (a.-c.)

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs.

3. Observe using graphs and tables that a quantity increasing exponentially exceeds a quantity increasing linearly, quadratically, or as a polynomial function.

Interpret expressions for functions in terms of the situation they model.

5. Interpret the parameters in a linear or exponential function in terms of a context.

6 Materials

- $\sim$ 100 M&Ms per group
- Graph paper
- Graphing calculator (optional)

7 Procedure

Time	Procedure
15 minutes	The teacher begins the lesson with any background information the students are lacking. This may include reviewing the slope-intercept form of linear equations and the standard form of exponential equations, techniques in recording data and graphing it properly, or familiarizing students with basic graphing on a calculator.
10 minutes	Initially, students will be presented with the standard equations for linear and exponential equations. They will have time to use a graphing calculator to consider many examples and make observations about how the different variables in the equations relate to the graph.
25 minutes	Then, students pour all of the M&Ms out and remove a given color, one color at a time till none are left, recording remaining number of M&Ms at every step. Students graph the results and define a linear function that fits the data.
25 minutes	Finally, students pour all of the M&Ms out and remove all M&Ms with "m" facing down. They record the number remaining and repeat until there are less then 10 M&Ms. Again, students graph the results and define an exponential function that fits the data.
15 minutes	After the activity is complete, a discussion will be ran by the teacher. During the discussion, students can compare their results as well as compare the differences between linear and exponential functions. They can then discuss how these different functions relate to real life instances, like the examples in the introduction.

The timeline of the lesson is highly dependent on the age and skill level of the students participating and should be adjusted accordingly.