Noʻa

Introduction

Noʻa was an ancient game played by two teams of people, either indoors on a mat or outdoors on a sandy beach. Playing this game requires five bundles of kapa to be placed between the two teams, a Noʻa (stone), and a maile or stick. Sometimes this game was played outdoors with piles of sand used in place of kapa.

The teams then decide who goes first; the selected team nominates someone on their team to hide the noʻa. This person takes the Noʻa in their hand, with their thumb holding it against the palm and fingers. With the back of their hand up and the noʻa hidden beneath, he/she places their hand in turn beneath each kapa or bundle of sand. They may also return to one or more of the piles multiple times, however the hiding process should not be prolonged too long. Sometime during this process he/she will release the Noʻa.

The opposing team will be watching this process in an attempt to see where the Noʻa has been released. The guessing team members then collaborate and their spokesman touches one of the piles with the maile. If he guesses right, they score a point. The Noʻa is then retrieved and the process is repeated with the teams swapping roles. The game continues until a predetermined score is reached.

Grade Levels and Topics

- **11-12: Probablility and Statistics**
  - Focus on counting, calculating the total number of combinations and permutations, independent and dependent events, and calculating probabilities.
- **11-12: Statistics**
  - Focus on the binomial distribution, using the normal distribution to approximate a binomial random variable, and hypothesis testing.
- **Probability:**
  - **Counting:** An efficient way of counting is necessary when dealing with statistical data. Common counting techniques are finding the total number of combinations or permutations with the use of factorials. The factorial symbol is ! where $n! = (n)(n-1)(n-2)\cdots(3)(2)(1)$. Note that $0! = 1$.
  - **Permutations:** An arrangement (or ordering) of a set of objects; in other words, if order does matter then it is a permutation. As an aside, combination locks should really be called permutation locks. Common notation for the number of permutations of n distinct objects taken r at a time is P_r^n, ${}_nP_r$, or ${}^nP_r$. Note $P_r^n = \dfrac{n!}{(n-r)!}$

- **Combinations:** A selection of all or part of a set of objects, without regard to the order in which objects are selected. If order of a selection of objects does not matter, then it is a combination. Common notation for the number of combinations from choosing r objects from n is ${}_nC_r$ or $\binom{n}{r}$.
- **Sample spaces:** set of all possible outcomes of an experiment. For example, the sample space of the outcome for a coin flip is $Heads, Tails$. Note that the sample space could also be $Heads, NoHeads$. The probability of all event in the sample space have to add up to one.
- **Independent events:** Two events, A and B, are independent if the fact that A occurs does not affect the probability of B occurring. Independent events have the following properties:
  - $P(A \text{ and } B) = P(A \cap B) = P(A)P(B)$
  - $P(A \text{ given } B) = P(A|B) = P(A)$
- **Dependent events:** Two events are dependent if the outcome or occurrence of the first affects the outcome or occurrence of the second so that the probability is changed.
- **Addition rule:** Used when finding the probability of the union of two events A, B. If two events are independent then $P(A \text{ or } B) = P(A \cup B) = P(A) + P(B)$. If two events are dependents then $P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \text{ and } B) = P(A) + P(B) - P(A \cap B)$.
- **Conditional probability:** The probability of an event A, given that another event B has already occurred. The notation used for the previous statement is $P(A|B)$. Note that $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$. If A and B are independent then that formula simplifies to $P(A|B) = P(A)$.
- **Compliment:** The Complement of an event is all outcomes that are NOT the event.

• **Statistics:**

- **Hypothesis testing:** Hypothesis testing, in general, is determining the probability that a certain hypothesis is true. While hypothesis testing, you assume the null hypothesis H_0, run an experiment to gather data, and test to see how likely you were to obtain that result given that the null hypothesis is true. To increase scientific validity in testing a hypothesis, the level of significance α is chosen prior to inputting data; α is commonly 0.1, 0.05, or 0.01. When rejecting the null hypothesis you are saying that the probability of me obtaining this data given that the null hypothesis was true is too low for me to keep the null hypothesis as true.
- **Using normal distribution to approximate binomial distribution:** When a hypothesis test is being done involving the binomial distribution it can be really difficult with a large sample size. To simplify the computation needed the normal distribution is used as an approximation. Note that this can only be done when $np > 5$ and $n(1-p) > 5$ where n is the sample size and p is the probability that you are testing.
- **ANOVA:** ANOVA is used when comparing three or more means (testing to see whether or not all the means are equal) at the α level of significance.
- **Tukey intervals/Tukey's method:** With Tukey's method, first you use ANOVA to determine that the means of samples are not equal. When the ANOVA shows that the means are not equal, with α level of significance, Tukey intervals are used to see which means are the culprit.

Objectives

- Students will see briefly how to apply probabilistic models.
- Students will gain an understanding of what a sample space is, the difference between independent and dependent events, and conditional probabilities through this example.
- Students will apply compliments in order to compute probabilities.
- Students will compute probabilities using the total number of permutations/combinations.
- Students will properly state the null hypothesis and alternate hypothesis and be able to test whether or not to reject the null hypothesis at some α level of significance.

Materials and Resources

- A Noʻa or small pebble
- Rod with a feather.
- Five bundles of kapa or cloth.
- Worksheets

Procedure

1. Create two teams of three or more.
2. Lay out the 5 bundles of kapa (or cloth squares) and number them 1 through 5.
3. Team 1 will nominate someone to hide the Noʻa and Team 2 will nominate a spokesman to choose where the noʻa was hidden.
4. Team 1 will hide the Noʻa while the people in Team 2 watch.
5. Team 2 will now collaborate on where they think the Noʻa was hidden then spokesman will use the rod to point out which kapa contains the Noʻa under it.
6. If Team 2 guesses correctly then Team 2 gets a point, if not then no one gets a point.
7. Keep track of which kapa the opposing Team placed the Noʻa under.
8. Now Team 1 and Team 2 will change roles.
9. Continue this process for at least 10 rounds.
10. The team with the most points at the end will be the winner or the first team to a predetermined amount of points will be the winner.
11. Students will then use that data to answer questions on the following worksheet.

Probability Discussion

- Discuss with the students examples of a dependent events and independent events.
  - Two events are **dependent** if the outcome or occurrence of the first affects the outcome or occurrence of the second so that the probability is changed.
  - Two events, A and B, are **independent** if the fact that A occurs does not affect the probability of B occurring.
- Discuss with the students how the **sample space** of an experiment is the set of all possible outcomes of an experiment. For example, ask the students what the sample space of a coin flip is and/or other games of chance.
- Probability is an interesting and diffucult subject to interpret. Discuss with the students what they think the probability of an event represents. For example, if the probability of flipping a heads is 50%, what does that 50% tell us? How can we use that information? etc...
- In this activity, students will justify whether or not cheating is involved mathematically. We all have an intuitive sense of whether or not someone is cheating, but through probability, we can quantify and justify that thought, whether it be through hypothesis testing or calculating the probability that an event occurred.
- A vital discussion to have with the students is that you should only define a probability to an event in the presence of randomness; for example look at the sequence $\{1, 2, 3, 1, 2, 3, 1, 2, 3, ...\}$, applying a probability to the next terms in the sequence is pointless as there is a set pattern in play (no randomness).

Statistics Discussion

- Discuss with the students that hypothesis testing is a powerful tool, however they should be on the lookout for data that may look too perfect. One example to discuss is if you have a sample of 50 games and you see that the rock was under each kapa 10 times then it will pass the hypothesis test that the probabilities are $\frac{1}{5}$, however it looks a bit to perfect, hence you may need to be a little skeptical of how the experiment was done.
- Discuss how the binomial distribution is usually approximated by the normal distribution when the sample size n is large enough. If $np > 5$ and $n(1-p) > 5$ then the binomial distribution can be approximated with the normal distribution which makes hypothesis testing a bit easier.

No‘a Activitiy Questions **Name:** ___________________________

Get into teams of 3 or more people and find another team to play at least 10 rounds of No‘a. Before starting, lay out 5 bundles of kapa (or something to hide the No‘a under). Then label each kapa 1 through 5. Before starting answer the following questions.

1. When your opponent hides the No‘a under one of the kapa, assuming that each kapa has an equal chance of having the No‘a under it, what is the probability of guessing correctly?

Now, use the space below to keep track of which number kapa your opponent placed the No‘a was under and to keep track of how many points you get. After collecting the data, answer the following questions with your team.

Probabilities

1. What was your teams score? What was the probability of your team obtaining that score?

2. What was your opponents score? What was the probability of them obtaining that score?

3. What is the probability that your two scores would add up to that number?

Statistics

4. Test whether or not your opponent had a better than a 50% chance of achieving their score at the $\alpha = 0.1$ level of significance.

5. Using your data, determine the probabilities of your opponents placing the No'a under each kapa. Are each close to the same amount? Were your opponents good at hiding the kappa (no patterns, random distribution, etc...)?

6. As a class, fill out the chart and determine whether or not the probability distribution for each kapa with the No'a underneath is the same. Calculate this distribution at the $\alpha = 0.01$ level of significance.

Kapa Number	Team 1	Team 2	Team 3	Team 4	Team 5	Team 6	Team 7	Team 8	Team 9	Team 10
1										
2										
3										
4										
5										

No‘a Worksheet: Probabilities **Name:** ______________________________

1. You are preparing to set-up a game of No‘a. While searching for kapa to hide the No‘a under, your friend tells you that he has a box which contains kapa of four different shapes, each shape has three colors, and each color has seven different patterns on them. How many different kapa does your friend have?

2. You are to lay out three circular and two square pieces of kapa in a row to prepare for the next game of No‘a. How many different ways are there to lay out the kapa?

3. You are hiding two No‘a, what are the total number of ways in which you can place them under each of 5 kapa given that they both cannot be under the same kapa? How many ways can this be done if both No‘a can be placed under the same kapa?

4. Determine if following events will be treated as independent or dependent:
   (a) Not locating the noʻa on the first guess but finding it on the second guess.
   (b) The probability of choosing wrong on the first guess and the probability of choosing correct on the second guess given that the game is reset after the first guess.
   (c) Given you only have one guess per game, what is the probability that you win three games in a row?

5. What is the sample space for this game?

6. State the compliment of the following events:
   (a) Finding a noʻa on the first guess.
   (b) Finding a noʻa on the first, third, or fifth guess.
   (c) You and your opponent end up with the same score.

For the following questions, consider making a probability tree to help visualize the outcomes. The probability tree provided may be useful for questions 7, 11, 12, and 13

7. What is the probability of correctly locating the no‘a given you have one chance? What would the probability be if you had two chances without the game begin reset on an incorrect guess? Three chances? Four chances? Five chances? Remember, the game is NOT reset if the no‘a is not located after a guess (no‘a is not hidden again).

8. Lets say you have five chances to guess where the no‘a is, but this time the game is reset when you've failed to find the no‘a what is the probability that you needed one guess to find the no‘a? Four guesses? Three guesses? Two guesses? Remember the game IS reset after the no‘a is located after a guess (opponents re-hide the no‘a).

9. You are now playing against one of your friends, you see in a reflection that he didn't place the No‘a under the two kapa that are at the ends, what is the probability you get it right on the first try given that you use this information to your advantage?

10. If there are now two No‘a hidden under different kapa, then what is the probability of finding one on the first guess and the other on your second guess? What about finding one on your first guess and finding the last one on the fifth guess? In both cases, the no‘a is NOT hidden again after each guess. Show all work.

11. You have 1 point and your opponent has 3 points; the winner will be the person with the greater number of points, a tie is equal points, and a loss is less points. If you have 4 more rounds of guessing left, and your opponent has 3 more left, then what is the probability that you and your opponent will tie? The probability that you lose? The probability that you win? Try playing this experiment out with a partner before calculating the probabilities.

12. If you were to play 20 rounds of No‘a what would your expected score be?

13. You and your opponent have a score of zero, what is the probability that after 5 rounds of guessing each that you both end up with a different score?

Challenge (Binomial Distribution):

1. It is obvious to see that if both your opponent and you have the same odds of guessing correctly, then the odds of beating your opponent is the same as your opponent beating you. However, what is the probability that you have more points than your opponent after 3 rounds given that your odds of gaining a point per round is $\frac{1}{5}$ and your opponents odds of gaining a point per round is $\frac{1}{4}$? Find the probability of tieing with your opponent and use the probability you win and the probability you tie to find the probability that your opponent will win.

No‘a Worksheet: Statisitcs **Name:** ____________________

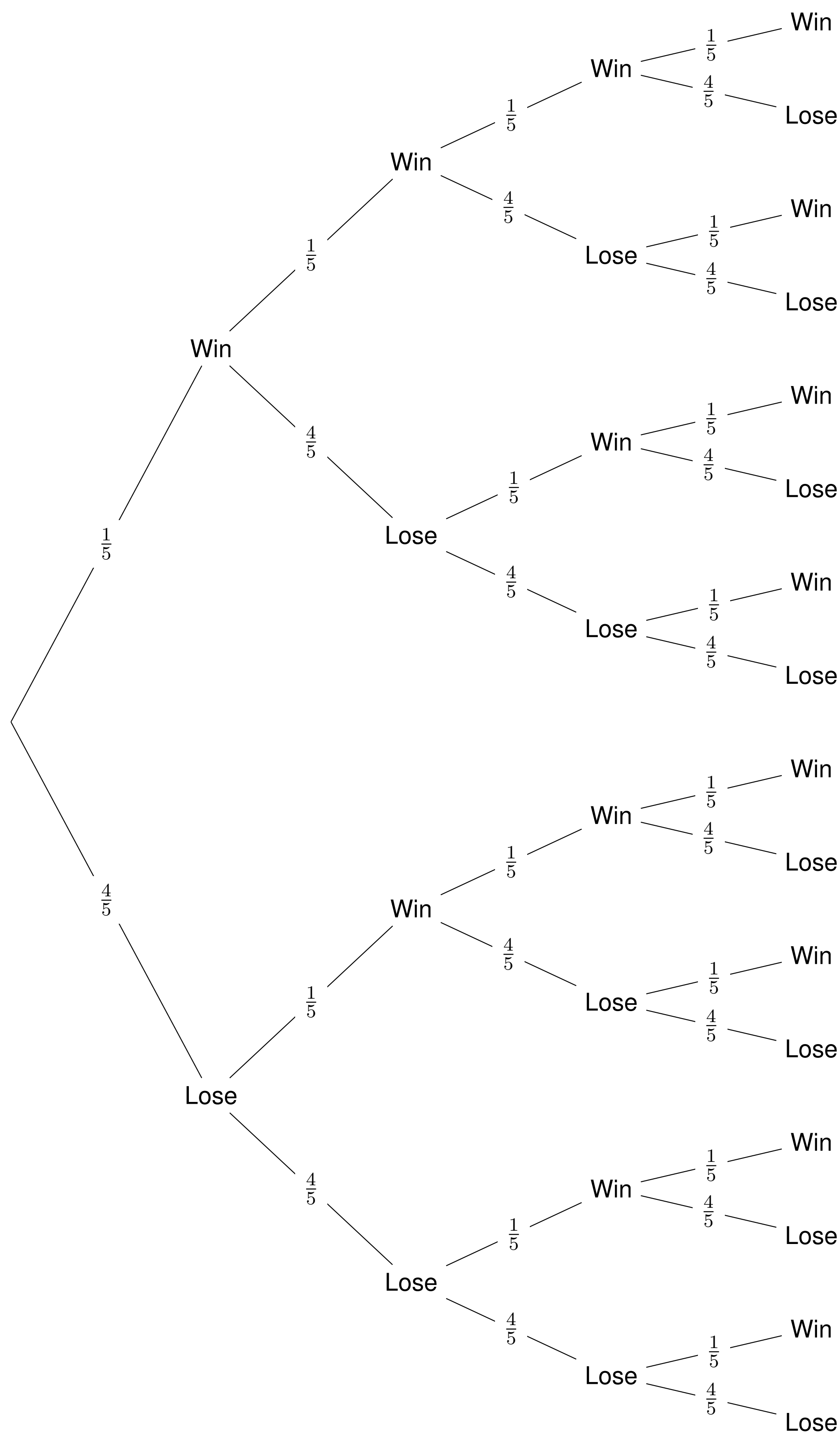
1. After playing 12 games of No‘a, the No‘a was under a specific kapa 5 times, we will call it kapa number 1. Test whether or not the probability of being under kapa number 1 is greater than $\frac{1}{5}$ at the $\alpha = 0.05$ level of significance. State a proper null hypothesis H_0, alternate hypothesis H_1, and decision rule. Then test whether or not you will reject or fail to reject H_0 at α level of significance. Use chart on the last page.

2. You suspect that your opponent may be someone who has been suspected of cheating in past games. If you are to play 16 rounds with him what score would he need to get in order for you to be 99% confident that he is doing more than just random guessing. Use the chart on the last page.

3. After playing 38 rounds of No‘a your opponents score was 11. Use the normal distribution to determine whether or not your opponent has better than a $\frac{1}{5}$ chance of getting a point per round at $\alpha = 0.1$ level of significance. Use chart on last page.

4. Use the following table of data to determine whether or not the probability of being under each kapa is the same at the $\alpha = 0.05$ level of significance.

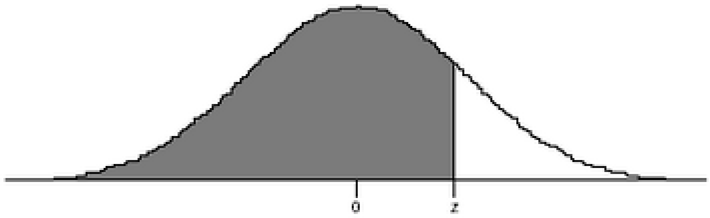
Kapa Number	Game 1	Game 2	Game 3	Game 4
1	0	1	0	0
2	6	5	5	3
3	2	1	1	3
4	0	2	4	2
5	2	1	0	2



Probability Tree for No‘a

This table shows the probability of r successes in n independent trials, each with probability of success *p*.

n	*r*	.01	.05	.10	.15	.20	.25	.30	.35
12	0	.886	.540	.282	.142	.069	.032	.014	.006
	1	.107	.341	.377	.301	.206	.127	.071	.037
	2	.006	.099	.230	.292	.283	.232	.168	.109
	3	.000	.017	.085	.172	.236	.258	.240	.195
	4	.000	.002	.021	.068	.133	.194	.231	.237
	5	.000	.000	.004	.019	.053	.103	.158	.204
	6	.000	.000	.000	.004	.016	.040	.079	.128
	7	.000	.000	.000	.001	.003	.011	.029	.059
	8	.000	.000	.000	.000	.001	.002	.008	.020
	9	.000	.000	.000	.000	.000	.000	.001	.005
	10	.000	.000	.000	.000	.000	.000	.000	.001
	11	.000	.000	.000	.000	.000	.000	.000	.000
	12	.000	.000	.000	.000	.000	.000	.000	.000
16	2	.010	.146	.275	.277	.211	.134	.073	.035
	3	.000	.036	.142	.229	.246	.208	.146	.089
	4	.000	.006	.051	.131	.200	.225	.204	.155
	5	.000	.001	.014	.056	.120	.180	.210	.201
	6	.000	.000	.003	.018	.055	.110	.165	.198
	7	.000	.000	.000	.005	.020	.052	.101	.152
	8	.000	.000	.000	.001	.006	.020	.049	.092
	9	.000	.000	.000	.000	.001	.006	.019	.044
	10	.000	.000	.000	.000	.000	.001	.006	.017
	11	.000	.000	.000	.000	.000	.000	.001	.005
	12	.000	.000	.000	.000	.000	.000	.000	.001
	13	.000	.000	.000	.000	.000	.000	.000	.000
	14	.000	.000	.000	.000	.000	.000	.000	.000
	15	.000	.000	.000	.000	.000	.000	.000	.000
	16	.000	.000	.000	.000	.000	.000	.000	.000



Normal Deviate z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-4.0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-3.9	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-3.8	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-3.7	.0001	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-3.6	.0002	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
-3.5	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483

Worksheet Answer Key

Probability Answers

1. $4 \cdot 3 \cdot 7 = 84$
2. $\binom{5}{3,2} = \frac{5!}{3!2!} = 10$
3. $\binom{5}{2} = 10$
   $\binom{5}{1}\binom{5}{1} = (5)(5) = \boxed{25}$. Note we used the multiplication rule since the event of placing one no'a and then another is independent from one another
4. (a) Dependent. the probability of choosing correct on the second guess is dependent on what happens in the first guess. $\left(\frac{4}{5}\right)\left(\frac{1}{4}\right) = \frac{1}{5}$
   (b) Independent. The game resetting after every first guess of the game, so choosing wrong on the first guess and choosing correct on the second guess is equivalent to saying that you choose wrong on the first guess of one game and choose correct the second game, since each game is obviously independent of each other these two events are independent. $\left(\frac{4}{5}\right)\left(\frac{1}{5}\right) = \frac{4}{25}$
   (c) Independent. Similar reasoning as (b). $\frac{1}{125}$
5. {no no'a, no'a}
6. (a) Finding the no'a on the second, third, fourth, or fifth guess.
   (b) Finding a no'a on the second or fourth guess.
   (c) You and your opponents scores are not equal.
7. $\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1$
8. $\frac{1}{5}$ is the probability for all.
9. $\frac{1}{3}$
10. $\frac{1}{10}$ is the probability for both.
11. $1 - P((0,0) \cup (1,1) \cup (2,2) \cup (3,3) \cup (4,4) \cup (5,5))$
    $= 1 - (P((0,0)) + P((1,1)) + P((2,2)) + P((3,3)) + P((4,4)) + P((5,5)))$
    $= 1 - \left(\left(\frac{1024}{3125}\right)^2 + \left(\frac{256}{625}\right)^2 + \left(\frac{128}{625}\right)^2 + \left(\frac{32}{625}\right)^2 + \left(\frac{4}{625}\right)^2 + \left(\frac{1}{3125}\right)^2\right)$
    $= 1 - \frac{3122577}{9765625} = \boxed{\frac{6643048}{9765625} \approx 0.680248}$

12. $P(\text{win}) = \boxed{\frac{1136}{78125} \approx 0.0145}$

    $P(\text{tie}) = \boxed{\frac{6924}{78125} \approx 0.0887}$

    $P(\text{lose}) = \boxed{\frac{70065}{78125} \approx 0.8968}$

13. 4

14. $P(\text{Win}) = \binom{3}{0}\left(\frac{3}{4}\right)^3\left(\sum_{n=1}^{3}\binom{3}{n}\left(\frac{4}{5}\right)^{3-n}\left(\frac{1}{5}\right)^n\right) + \binom{3}{1}\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right)\left(\sum_{n=2}^{3}\binom{3}{n}\left(\frac{4}{5}\right)^{3-n}\left(\frac{1}{5}\right)^n\right)$

    $+\binom{3}{2}\left(\frac{3}{4}\right)\left(\frac{1}{4}\right)^2\left(\binom{3}{3}\left(\frac{1}{5}\right)^3\right) = \boxed{\frac{2007}{8000} = 0.250875}$

    $P(\text{Tie}) = \sum_{n=2}^{3}\binom{3}{n}^2\left(\frac{1}{4}\right)^n\left(\frac{3}{4}\right)^{3-n}\left(\frac{1}{5}\right)^n\left(\frac{4}{5}\right)^{3-n} = \boxed{\frac{3133}{8000} = 0.391625}$

    $P(\text{Lose}) = 1 - (P(\text{Win}) + P(\text{Lose})) = \boxed{\frac{2860}{8000} = 0.3575}$

Statistics Answers

1. Let X be the number of times that the No‘a was under kapa number 1 during 12 games. Note $X \sim B(12, p)$.
   $H_0 : p = \frac{1}{5}$
   $H_1 : p > \frac{1}{5}$
   Decision rule: Reject H_0 if $P(X \geq 5) < 0.05$.
   $P(X \geq 5) = 0.0327934976 < 0.05 = \alpha$. Reject H_0 at $\alpha = 0.05$ level of significance.

2. Let X be the number of points the opponent scores after 16 rounds of No‘a.
   Note $X \sim B(16, p)$.
   $H_0 : p = \frac{1}{5}$
   $H_1 : p > \frac{1}{5}$
   Reject H_0 if $P(X \geq c) < \alpha = 0.01$. From the chart we see that $P(X \geq 7) > 0.01$ but $P(X \geq 8) < 0.01$. Hence we will reject H_0 at $\alpha = 0.01$ level of significance if the opponent gets a score of eight or higher. This tells us that we will be 99% confident that our opponent is doing something other than random guessing.

3. Let X be the number of points gained by opponent from 38 rounds of No‘a. Note $X \sim B(38, p)$.
   $H_0 : p = \frac{1}{5}$
   $H_1 : p > \frac{1}{5}$
   Note that $38 * \frac{1}{5} = 7.6 > 5$ and $38(1 - \frac{1}{5}) = 30.4 > 5$ hence we can use the normal distribution to approximate $B(38, p)$.
   Decision rule: Reject H_0 if $z > Z_{.9} = 1.28$.

$z = \frac{x - np}{\sqrt{np(1-p)}} = 1.3789 > 1.28 = Z_{.9}$. Hence we reject H_0 at the $\alpha = 0.1$ level of significance. This tells us that our opponent does indeed have a better than $\frac{1}{5}$ chance of gaining a point per round of guessing.

4.

Source of Variation	d.f.	SS	MS	F_0
Between groups	4	SSA=45	MSA=11.25	8.8816
Error	15	SSE=19	MSE=1.2667	
Total	19	SST=64		

$\mu_1 - \mu_2$	-4.5	(-6.957, -2.043)	Reject
$\mu_1 - \mu_3$	-1.5	(-3.957, 0.958)	NS
$\mu_1 - \mu_4$	-1.75	(-4.207, 0.708)	NS
$\mu_1 - \mu_5$	-1	(-3.457, 1.457)	NS
$\mu_2 - \mu_3$	3	(0.543, 5.457)	Reject
$\mu_2 - \mu_4$	2.75	(0.293, 5.207)	Reject
$\mu_2 - \mu_5$	3.5	(1.043, 5.957)	Reject
$\mu_3 - \mu_4$	-0.25	(-3.233, 2.233)	NS
$\mu_3 - \mu_5$	0.5	(-1.957, 2.957)	NS
$\mu_4 - \mu_5$	0.75	(-1.707, 3.207)	NS