Operations Research

Grade Levels

This activity is intended for 9th-12th grade students.

Objectives and Topics

In this activity, students learn to apply mathematics creatively, strategically, and logically with actually experiencing and attempting to answer difficult real-world problems.

Introduction and Outline

Operations Research (OR) is the branch of applied mathematics which seeks to optimize efficiency in a wide-range of real-world fields, from business, to civil engineering, to transportation, to event planning, to project management.

My philosophy of applied mathematics is that it is always easy to come up with a solution, but it is difficult to come up with an efficient solution. Indeed, I see the entire reason behind mathematics as a way to efficiently approach problems. Given an algebra problem, it is easy to find a solution: Guess and check until you find something that works. But if you know some math, you can find a correct answer in a far more efficient way. Similarly, one could launch rockets into space randomly until one reaches the moon - but use of math and science make the process a lot faster. Math makes possible what is otherwise practically impossible or simply impractical. The five problems on the worksheet below are in the same vein. It is easy to find a solution that works. It is difficult is to find an efficient solution.

Problems are designed for students to work in groups (2-4 students) so that they may discuss/debate what the "best" solution is. A class of students can be split up to work on different problems and then present to each other. Or groups can work on the same problem and present their individual solutions to the class and discuss differences (some problems will have greater variety than others).

There are no correct answers - but there are answers which are better than others. It is up to the students to prove to themselves (and their classmates) that their solutions is the best.

As a side-note, there are mathematical methods to finding the optimal solution for all of these problems - but the algorithms used are beyond the scope of most high school math classes (look up "simplex algorithm" as one example). My aim is that students are able to defend their choices with sound reasoning, not rigorous mathematical proof.

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In answering the questions, encourage your students to use their real-world experiences (because these are real-world problems) and common sense. Students should think about strategies in approaching the problems and be able to explain to each other what that strategy is and why it works.

Problems are split into several parts. The first part introduces the mathematical idea of the OR problem type with a "simple" question. The second part poses a real-world problem and asks students to define what it means for a solution to be "optimal". The last part takes the problem to a much more difficult level to analyze and will require careful thought, consideration, strategy, and perserverance.

Notes for Particular Problems

1) Think of parts a and b as a road network between cities (or airport hub network). A set of paths that connects all the points simply means can I get from one city to any other city (not necessarily directly). One strategy is to start at A and decide which one of the three paths (to B, C, D) to keep. Then move to B and choose which path of its four paths to keep. And so on. When fashioning a road network for Oahu, the tendency at first is to just draw around the perimeter of the island (much like our existing road network). But pose real-world situations: If I live in Kaneohe and I need to get to the airport, do I really want to drive all the way around the eastern tip of the island? This frequently prompts students to draw in additional roads (the H3!) that won't minimize length - but do improve efficiency.

   It's nice to have an actual roadmap of Oahu so that students can compare their road network to what actually exists - and what this says about the "common sense" of Oahu's road network.

2) This problem is the most mathematical and can be solved and proven using a lot of basic algebra. For any given dealership, students can calculate the cost for each factory of production and shipping, and then choose the cheapest option. Similarly, when constructing a new factory, students can simply calculate the change to costs and compare. This can be a tedious process. Before students do any calculations, they may wish to discuss what would make sense (I've made shipping costs relative to distance shipped) and quickly narrow down possibilities with estimation.

3) My map goes from Waipahu to downtown because I was partnered with Waipahu. I would suggest creating your own map (you can do it by hand!) from your school to various places on the island. Ideas of "best" can vary: As a mathematician, I always think to minimize distance or time. But some people want to maximize simplicity of the drive (straight lines and right turns are better). Adding traffic is an important factor that every driver takes into consideration when they think about how to get somewhere and when to leave. Something I have not done, but which you could add, are one-way streets (downtown is full of them). The more complicated and interconnected you make your network, the more options your students will have to analyze and choose from (and that's a good thing). Super-M Fellow John Rader created an activity called "Controlled Navigation" which inspired this problem. His lesson plan can be found on the Super-M website.

4) It is easy to find a solution which maximizes business - but it will be at great expense to the environment. One's emotions and personal values - how much one is willing to sacrifice - will affect the solution. Students should be encouraged to decide early-on what they consider important (business, environment, research, or culture). Mathematicians use these considerations in creating a cost function (see part 3). Students may

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choose to use their own cost function (or none at all) when answering part 4. Again - there is no correct answer, but students should be asked to justify their choice of what was ultimately important to them and why they chose the projects they did.

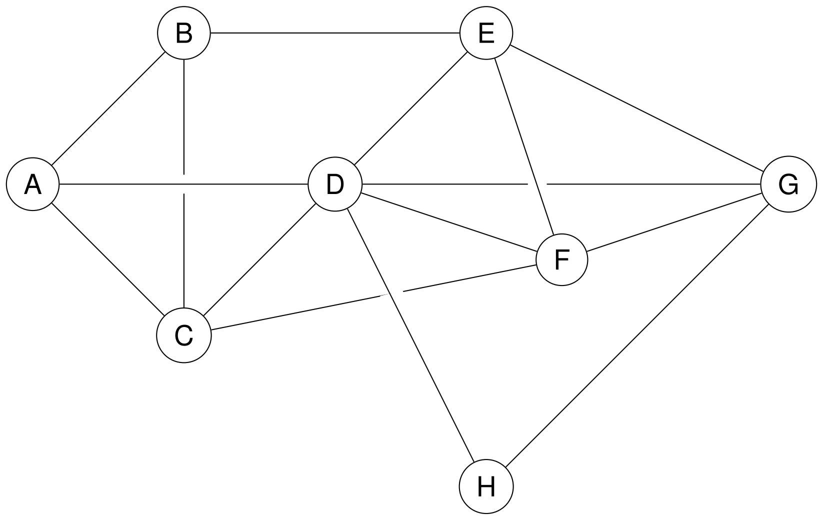
5) I created problem five because there were students in my class who wanted to be beauticians and saw no use for mathematics. My response is that mathematics may not be necessary for your job - but it will make you better/faster/more efficient/more profitable at it. In this case, a strategy is necessary to develop an effective business - and recognizing, creating, and implementing strategy is what math is all about. Something to keep in mind is that not all clients need to be served. But denying service has an effect on your business, not just because you lose the customer, but customers talk and gossip and advertise positively or negatively based on how they are treated. You want repeat customers because they give you a steady income. Students should consider their quality of life and what they want: do they want to work 7 days a week? Do they want a lunch break? Do they want to be home by 6 pm?

Operations Research Worksheet **Name:** ______________________________

Problem 1: Least Spanning Tree Problem

Given a set of points, which path connecting all the points has the minimum total distance?

1) In the figure below, 8 nodes have been marked A-G. Only certain paths exist between the nodes. The table gives the length of each path.

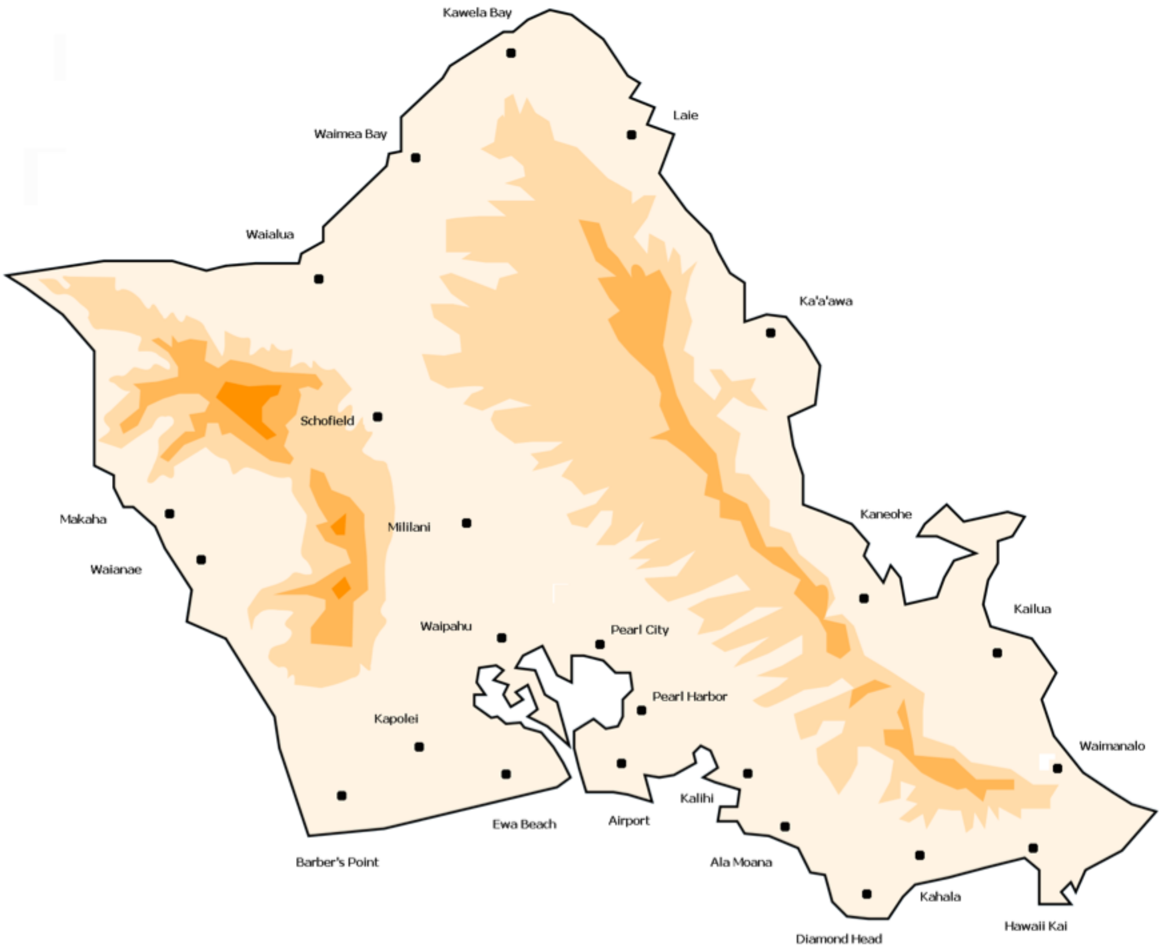


	A	B	C	D	E	F	G	H
A		4	3	5				
B	4		5	4	5			
C	3	5		3		5		
D	5	4	3		3	2	4	4
E		5		3		4	4	
F			5	2	4		2	
G				4	4	2		6
H				4			6	

a) Is there a set of paths that connects all the points? Are there multiple such paths?

b) Find a set of paths that connects all the points AND has minimal total length.

2) Below is a map of Oahu, with specific locations noted. Feel free to add your own locations, if you wish. Design a road network that will connect all of the locations together.

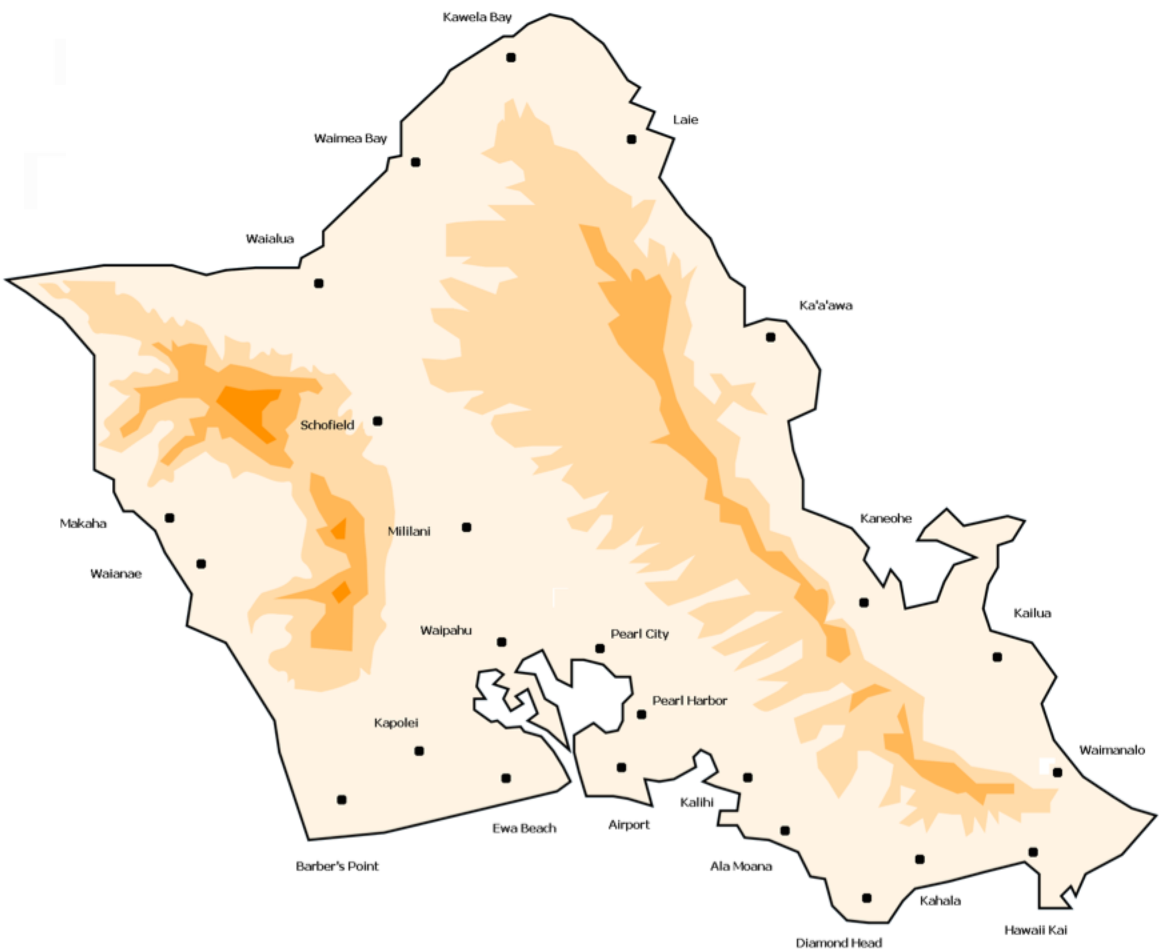


- Roads cost time and money. The longer your road, the more it will cost to construct.
- Bridges and tunnels cost more than roads to construct.
- Roads are meant to get people where they want to be efficiently.
- Intersections slow down travel. But intersections also allow more flexibility in routes.
- Traffic on Oahu is heavy in certain areas. Multiple routes might ease congestion.
- Ship access to Pearl Harbor must be maintained - no bridges in that area (but you can build them elsewhere).

3) Is your designed road network "optimal"? How do you define "optimal"? Is your network unique? (ie. Is there another network which is equally optimal?)

4) Redesign your road network, but now follow two specific rules:

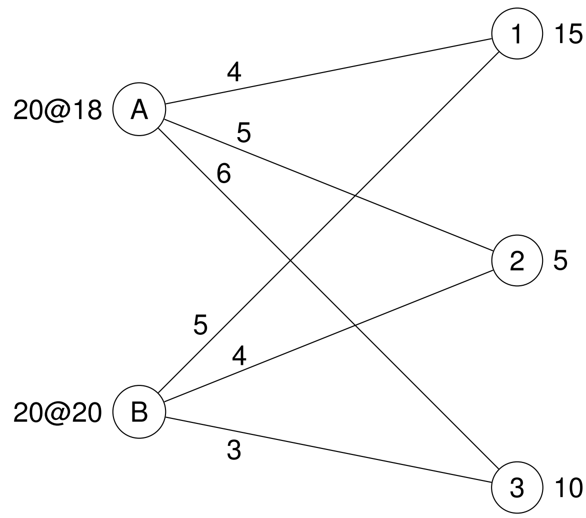
   - Tsunami and Natural Disaster Planning requires every location to have at least 2 ways in and out of the area. (Why? Fresh water and food are two of the most important resources to have in the event of a natural disaster. They must also be available in a relatively short time after the disaster. One of the major difficulties of disaster recovery is getting aid to those who need it. Fallen trees, landslides, flooding, and destroyed roads can isolate remote populated areas. Multiple routes to a location can reduce the chances of such isolation. Hawaii is especially vulnerable since many populated areas are along the coastline, and tsunamis and hurricanes are a threat.)
   - Tunneling through the mountains (the dark shaded part of the map) may now only occur at designated tunneling sites. (Why? Safe tunneling requires many geological conditions to be met or the tunnel will simply collapse. Moreover, Hawaii has many culturally sacred sites that are protected.)



Problem 2: Transportation Problem

Given a supply of materials, how do you transport it to where it's needed in the most efficient manner?

1) The figure below charts the following business situation: Two factories, A and B, which produce products and three retail sites. Each factory can only produce 20 units per day, but factory A has cheaper production costs (18 dollars/unit) than factory B (20 dollars/unit). The retail sites require 15, 5, and 10 units per day respectively. Shipping costs from each factory to each retail site are in the diagram.

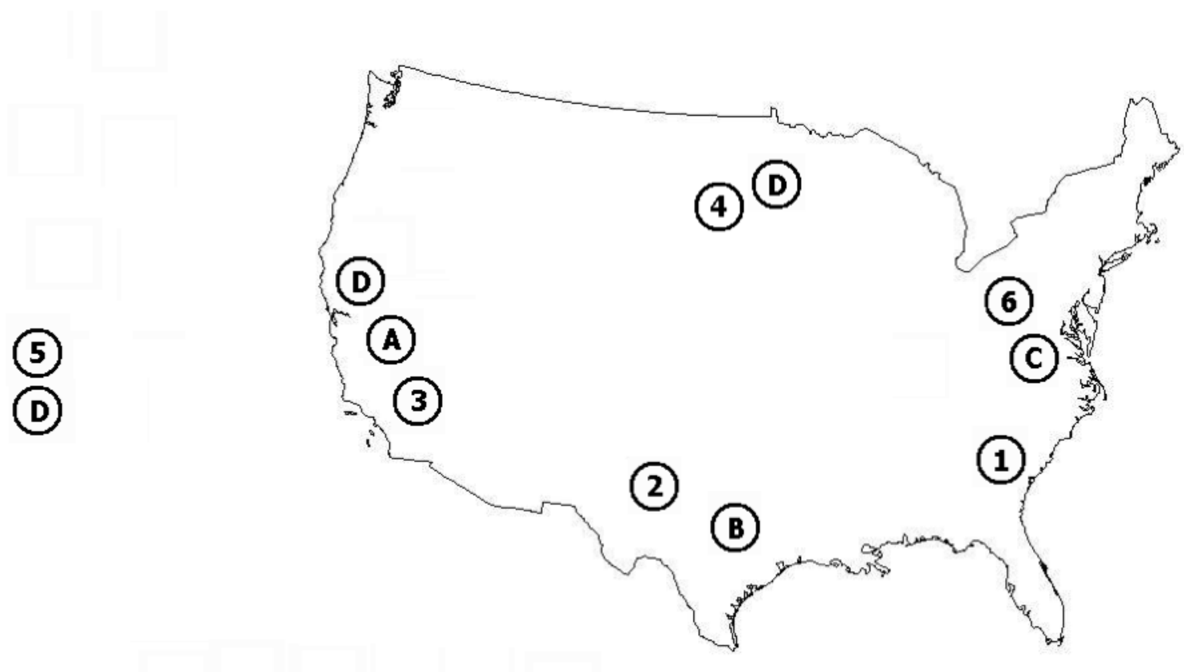


   a) Is there a business strategy that allows the retail requirements to be met? (ie. How much should each factory produce, and where should they ship it so that each retail site gets its required number of products?) Are there multiple such strategies?

   b) Find the strategy that meets the retail requirements and minimizes the costs. Is this strategy unique?

2) You are in charge of business strategy for a car company. Three factories in the US (A, B, C) produce car parts that need to be shipped to 6 dealerships (ignore factory D for now). Labor costs in different parts of the country mean that production costs differ at each factory. Each dealership requires a certain number of parts to meet demand (for repairs, replacements, etc). Shippings costs increase based on distance from factory to dealership. Details for the factories and the dealerships are in the tables below. Determine a business strategy for the situation.

Factory	Cost per Unit	Maximum Production Capacity
A	42	50
B	38	40
C	40	60



Dealership	Demand	Shipping Costs		
		A	B	C
1	25	8	4	1
2	20	2	1	5
3	30	2	4	5
4	15	4	3	3
5	25	10	15	20
6	20	8	5	2

Some things to consider:

- It's a waste to produce units you won't ship - you don't have to maximize production everywhere.
- Dealership 5 located in Hawaii has especially high shipping costs because parts must be shipped by air.
- Instead of guessing randomly, try choosing a plan that works and then tweaking it so that it gets better.

3) Is your business strategy optimal? How do you define "optimal"? What is the total cost of your strategy?

4) Your car company is looking into building a fourth factory. The factory will be small, capable of producing only a maximum of 20 units. Three sites are proposed (the D's on the map). The details are in the tables below. Which site is best? How will it affect your strategy? What will the new total cost be?

Site: Hawai'i
Production Cost: 45

Dealership	Shipping Costs
1	21
2	16
3	12
4	17
5	1
6	20

Site: West Coast
Production Cost: 42

Dealership	Shipping Costs
1	8
2	3
3	2
4	3
5	10
6	7

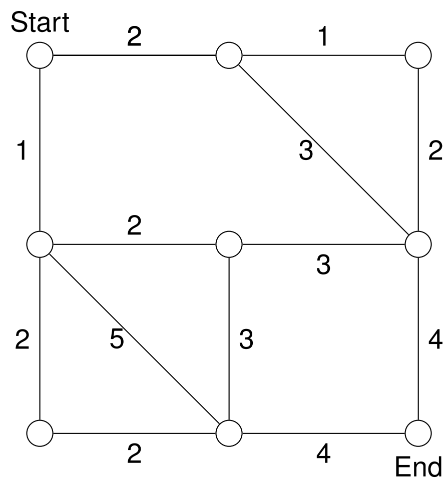
Site: Mid-West
Production Cost: 35

Dealership	Shipping Costs
1	4
2	3
3	4
4	1
5	15
6	4

Problem 3: Shortest Path Problem

How do you navigate a grid with minimal cost?

1) Below is a road grid. Each road has a cost associated with it. You may consider this cost to represent either distance or time (or whatever other value that makes sense to you).



a) Is there a path from the start to the end? Are there multiple such paths?

b) Find the path of minimal cost. Is it unique?

2) On the next page is a simplified road network of Oahu from Waipahu to Downtown Honolulu. Assume each path is length 1.

a) If you start at Waipahu, what is the best path to reach office building O in Downtown? How do you define "best"?

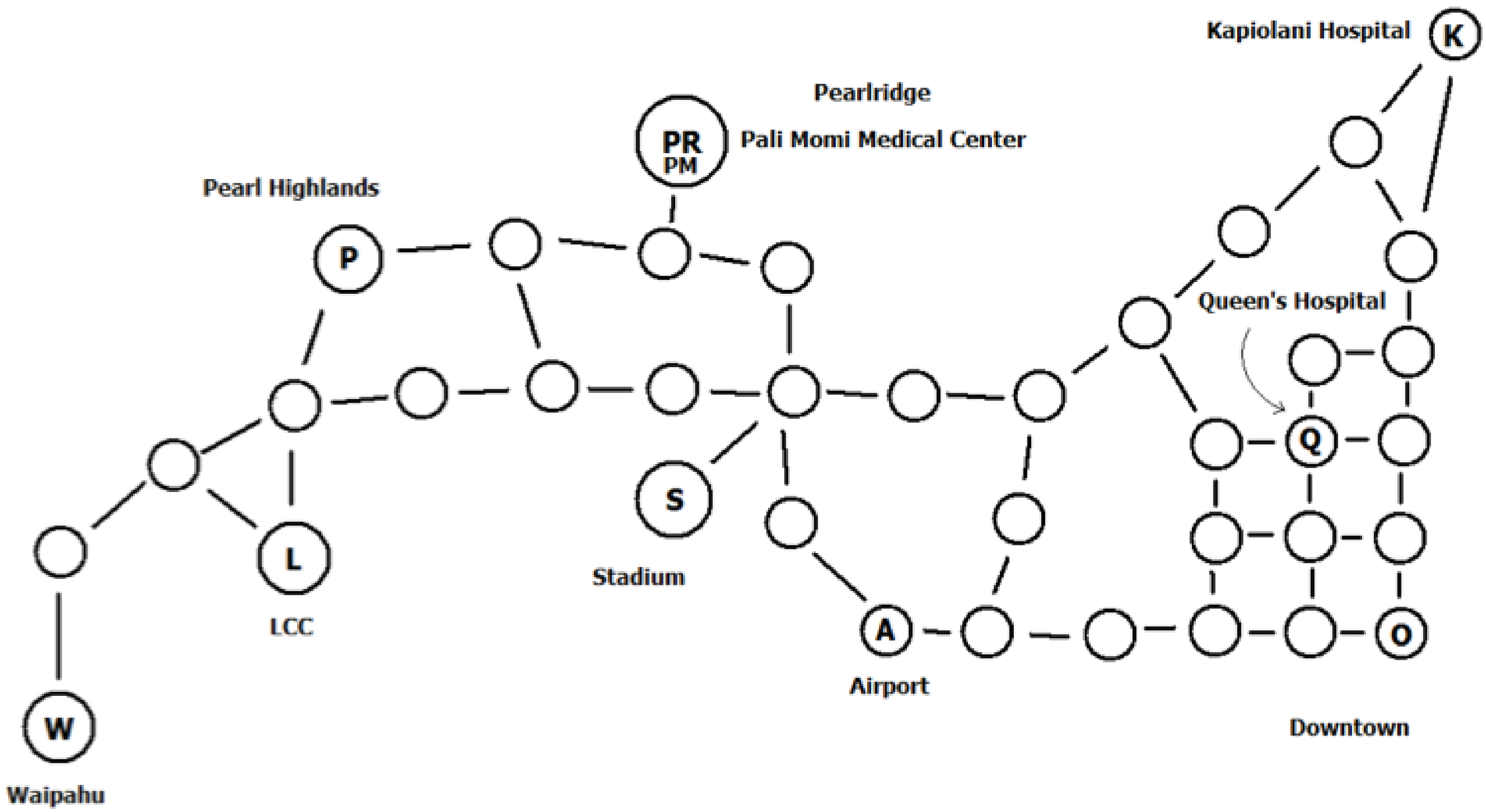
b) If you start at Waipahu, what is the nearest Hospital (Pali Momi, Queen's, or Kapiolani)? What is the shortest path to get there?

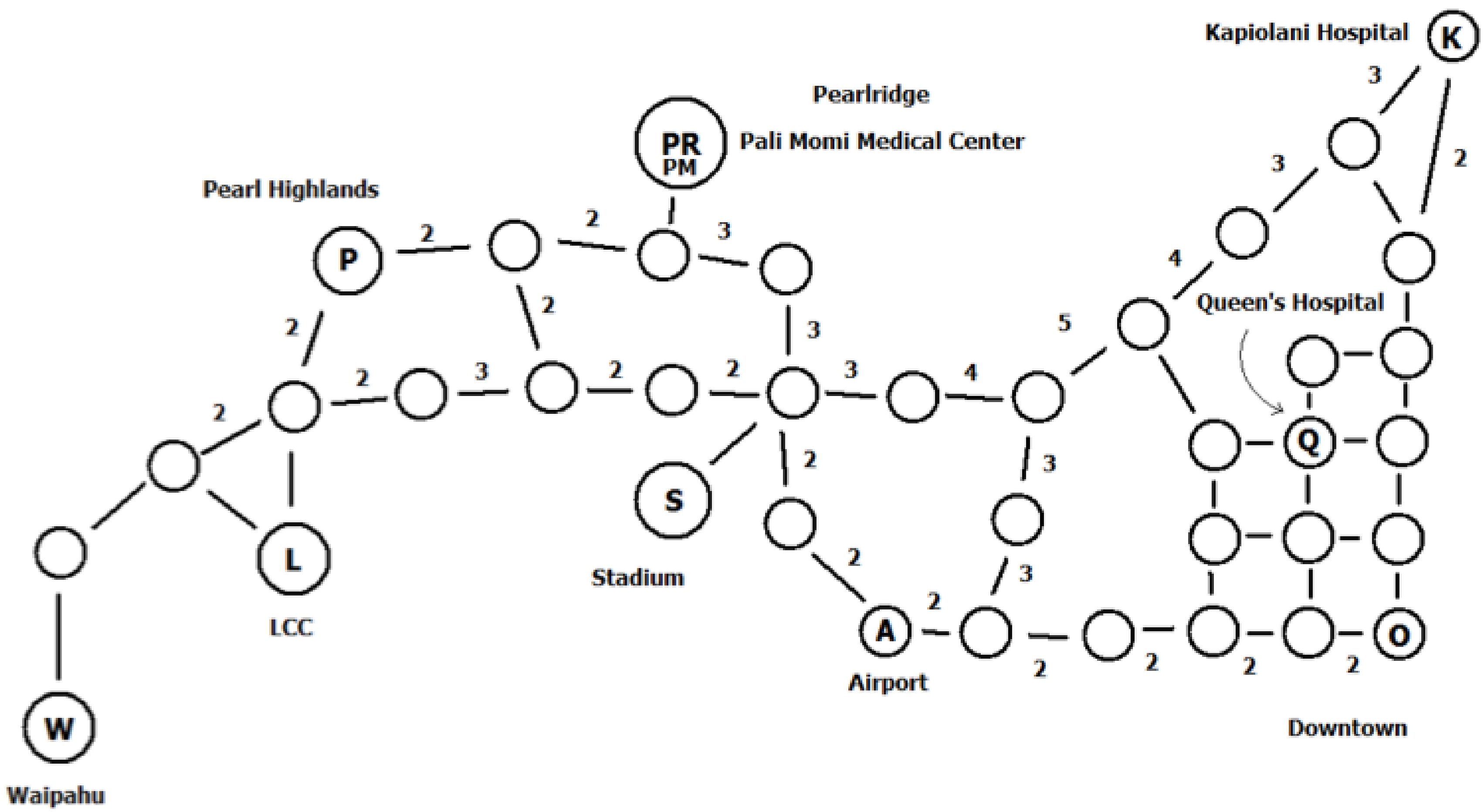
c) If you start at the airport, what is the nearest Hospital? What is the shortest path to get there?

3) Oahu's road systems are congested at different times of day. Consider the same road network at 4:30 pm on a weekday (second map). The path lengths are as indicated and now represent time (unmarked paths are still length 1). Heavily-congested roads will take longer to traverse.

a) If you start at Waipahu, what is the best path to reach office building O in Downtown? How do you define "best"?

b) If you start at the airport, what is the nearest Hospital? What is the shortest path to get there?


Kapiolani Hospital
K
Pearlridge
PR
PM
Pali Momi Medical Center
Pearl Highlands
P
Queen's Hospital
Q
S
Stadium
L
LCC
A
Airport
O
Downtown
W
Waipahu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Waipahu

4) The main post-office on the island is at the airport. This office receives all mail that comes in by air and must distribute it to all of the post-office branches on the island. Determine an optimal strategy for distributing this mail. Here are some things to consider:

- How do you define "optimal"?
- Every corner of the island has to receive mail service. For our example, however, simply ensure that every labeled site (hospitals, malls, schools, offices, etc) receives its mail.
- You have multiple mail-trucks available to you. But each mail-truck costs money for fuel, maintenance, and to hire the driver.
- Mail must arrive in a timely fashion. A single driver won't be able to do it all in one day.
- Traffic is always a concern - but traffic isn't always as terrible as it is at 4:30 pm. Use what you know of Oahu traffic.
- Mailtrucks have to return to the post office.

Problem 4: Assignment Problem

How do you assign agents to specific tasks in the most efficient way?

1) You are a project manager with four employees (1 through 4). You have four tasks to accomplish (A through D), and each worker can complete a task in a certain amount of time (see table). Assign your four employees so that all four tasks are accomplished most efficiently (lower numbers are better). Each worker must be assigned to one and only one task. Is your solution unique? (ie. Is there another way to assign the workers which is equally efficient?)

Workers and Task	A	B	C	D
1	4	6	5	4
2	5	3	2	4
3	3	3	5	4
4	5	4	6	4

2) You are a government official in charge of funding 10 different projects. You have $1,000,000 to distribute among the projects. Project details are below. Find an assignment strategy.

Project	Cost (in thousands)	Business/Finance	Env. Impact	Research	Culture
A	275	10	5	0	0
B	275	7	4	0	0
C	125	6	4	0	0
D	250	5	3	2	1
E	100	3	4	2	7
F	150	2	2	4	6
G	125	2	2	7	3
H	50	1	1	3	0
I	350	0	0	10	2
J	300	2	0	5	10

- Not every project needs to be funded.
- The Business/Finance score represents how much money and economic growth the project will bring to the island. Projects with high Business/Finance scores include road/housing/factory construction, and land development and improvement. These projects tend to be relatively cheap due to being funded by corpora- tions and often being able to pay for themselves in time. They are, however, hard on the environment.
- The Environmental Impact score represents how destructive the project will be to the ecosystem. The higher the score, the more pollution and environmental damage will occur.
- The Research score represents academic contributions generated by the project. These projects include ocean research, robotics, ecology, and educational programs in K-12 and university level.
- The Cultural score represents projects which seek to promote the culture of the island. This includes programs focused on Hawaiian/Pacific cultural immersion, fairs, archaeological research, and museum exhibits.

3) a) Is your assignment strategy optimal? How did you define “optimal”?

b) Mathematicians use what we call a cost function to determine optimality. Consider the cost function:

$$2 \cdot (\text{Business}) - \text{Env. Impact} + \text{Research} + 3 \cdot (\text{Culture})$$

What assignment strategy maximizes this function?

4) If partial-funding is allowed with equivalent partial-results, how does your strategy change? For example, you can now fund project F with 75 thousand dollars (exactly half the stated cost) for 1 Business, 1 Env. Impact, 2 Research, and 3 Culture (exactly half the stated values).

Problem 5: Scheduling Problem
How do you schedule your time and business?

1) You are a cosmetologist (or beautician or dental hygienist, etc). Suppose you have fifty clients a week, and each session takes 30 minutes. Plan a schedule for your week. Some things to consider:
    - Your clients have a fair chance to arrive up to 15 minutes late.
    - You need to schedule times at sane hours that people will actually be able to come in for.
    - You will probably want to take a break or two during your day.
2) Now we make things a little more interesting. Reschedule with the following requirements:
    - Clients 1-15 are just as before. They will take any reasonable 30 minute time slot.
    - Clients 16-20 will need a full hour at any reasonable time slot.
    - Clients 21 and 22 are brides-to-be and will each need two hours of your undivided attention on Saturday and Sunday, respectively.
    - Clients 23-25 are a family and will need to be scheduled one after another (30 minutes each) in the afternoon.
    - Clients 26-30 can only come in during weekday evenings (after 5 pm)
    - Clients 31-40 can only come in during the weekend.
    - Clients 41-45 can only come in during the morning (before 11 am) on weekdays or anytime on the weekends.
    - Clients 46-50 are regulars. They will come in (for 30 minutes) at: Monday 11:30 am, Tuesday 3:30 pm, Wednesday 9:30 am, Thursday 10:30 am, Friday 2:30 pm.
3) Now we make it even more interesting. You now have two employees (for a total of 3 workers), and 120 clients. Create a schedule for all three of you with the following requirements:
    - Clients 1-30 will take any 30 minute period at a reasonable time.
    - Clients 31-45 will need a full hour but will come in at any reasonable time.
    - Clients 46-50 are a bridal party and will need to be scheduled in the same block of time (between the three workers) for an hour each on Saturday morning (before 10 am).
    - Clients 51-55 are another bridal party and will need to be scheduled for an hour each on Sunday afternoon (after 12:00 pm, before 4:00 pm).
    - Clients 56-65 are all attending prom and will need an hour each on Friday afternoon (after 1:00 pm, before 5:00 pm).
    - Clients 66-80 can only come in during the weekends.
    - Clients 81-90 can only come in during weekday evenings (after 5 pm)
    - Clients 91-100 can only come in during the morning on weekdays (before 11 am) or anytime on the weekends.
    - Clients 101-103 are a family and must be scheduled at the same time (one worker each) during the afternoon (after 12:00 pm, before 5:00 pm) on any day. ?
    - Clients 104-106 are another family with the same requirements.
    - Clients 107-120 can only come in Monday-Thursday.