Salt Pouring

Grade Levels

Intended for grades K–12.

Objectives and Topics

The purpose of this lesson is to explore a physical phenomenon that results in geometric figures; students will be exposed to geometry, engineering, and scientific observation.

Introduction

This activity can range from a simple hands-on exploration for younger students to a full lab for geometry and algebra II students. Many do not see this physical phenomenon occurring in the world around them until reaching the collegiate level. This activity may take 1-2 periods, depending on the grade level.

Materials and Resources

- Several Basins or Containers
- Iodized Salt (or sand)
- Small Cups
- Index Cards (Or any sturdy card stock)
- Scissors
- Single Hole Punchers
- Handouts (see below)

Procedure

Cut out a shape from the index cards and place it atop the cups as diagrammed below so that it is elevated and balanced. Slowly pour the salt onto the shape. A 3D figure will form. Salt that falls off the shape will fall into the container or the cups for easy reuse.

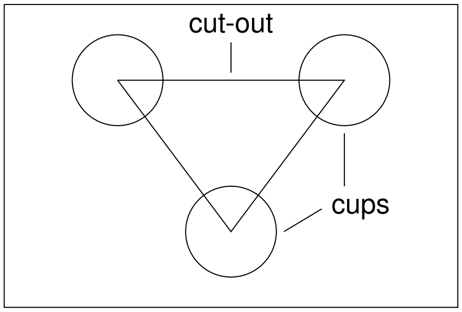
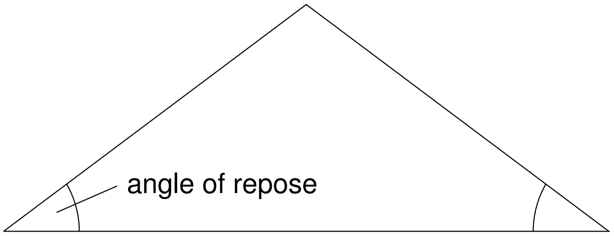


Diagram of shape cut out resting atop of the cups

The Science

The key to what is going on is what engineers call the "angle of repose." This is the critical angle at which granular substances (like salt, sand, or gravel) begin to slide down. Every substance has its own angle of repose. So at every edge of the shape, we form slopes at a very specific angle.

***Lots of interesting information on the angle of repose can be found on its Wikipedia page.

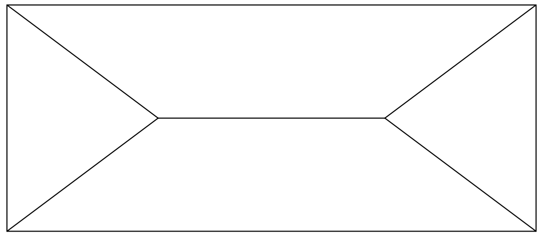


The Math

Since the salt has a specific angle of repose that it will take, the figures formed are always nice and neat.

If we pour atop a circle (or just out on the ground), we always get a conical shape (which should be predictable and intuitive). But note that, as in the picture above, if we cut the cone in half, we would have a face that is an isosceles triangle - which always has two congruent angles (the angles of repose!). Moreover, if we rotate an isosceles triangle around its line of symmetry, we form a cone. So at every point along the edge of the circle, we have the same angle of repose, which naturally forms a cone.

The ridge lines form where the slopes meet are of greatest interest to us. For a cone, all the slopes meet at one point, so there are no ridge lines. If you pour atop a triangle or quadrilateral, several nice ridge lines form that seem to connect the corners of the shape. Focus on just one of the corners. Notice that the ridge line bisects the angle of the corner. If we think about the isosceles triangles again, this should not be a great surprise.

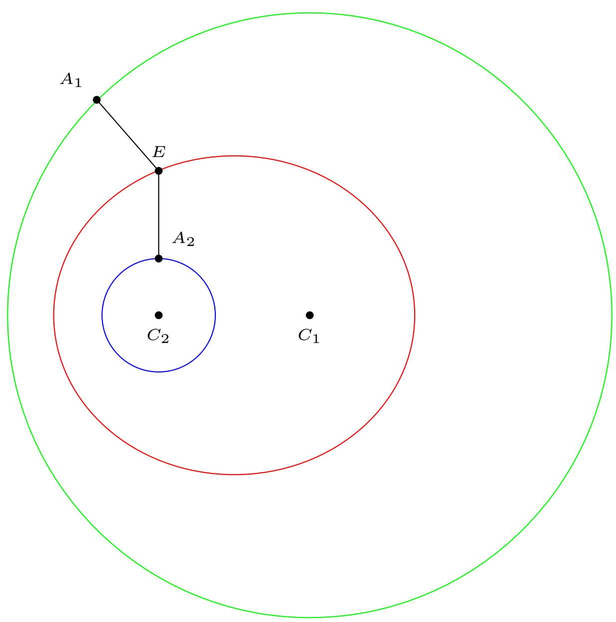


Pouring salt on a quadrilateral

Notice that the ridge lines are formed exactly in the middle of the closest edges of the shape. In mathematical terms, every point on the ridge line is equidistant from the edges. As more edges are introduced, the results become more complicated.

If salt is poured over a circle with a hole punched into it, we get what looks a lot like a volcano. In fact, when a real volcano erupts, the earth settles according to the angle of repose. However, the ridge line is an ellipse!

Here's a quick proof: Consider two circles, one contained in the other, with centers C_1 and C_2, and radii r_1 and r_2, respectively. Trace the locus of points equidistant between the two circles (diagrammed in red – and exactly where the ridge line will form), and choose any point, E, in this set.



Then:

$$\begin{aligned} C_1E + C_2E &= C_1E + C_2A_2 + A_2E \\ &= C_1E + C_2A_2 + A_1E \\ &= C_1E + A_1E + C_2A_2 \\ &= r_1 + r_2 \end{aligned}$$

Note: A_1E is perpendicular to the green circle (because that's how distance from point to curve is measured), so it will pass through C_1, allowing us to make the substitution $C_1E + A_1E = r_1$.

The radii are constant; therefore, every point E has a fixed total distance from C_1 and C_2, which defines an ellipse.

Finally, if we pour salt over three punched holes, we get the perpendicular bisectors of edges of the triangle defined by the three holes. The perpendicular bisectors meet at a point called the circumcenter, so named because if you were to circumscribe the triangle, the center of the circle would be at the circumcenter.

Other Notes of Salt Pouring

1. The larger the base shape, the more impressive the results will be, but more salt will be necessary.
2. Because salt is so pure and white, seeing the ridge lines can be difficult. Directed light will cast shadows that make viewing easier. If the room has multiple lights, try turning one off.
3. Older students in Geometry and Algebra II may be challenged to discover that the ridge lines are angle bisectors, perpendicular bisectors, and ellipses (or proving that it is an ellipse). How much of the above mathematics chosen to be revealed to students, as well as how much we ask them to reason/conjecture/prove on their own is up to us.
4. Younger students may simply enjoy pouring the salt (or sand if there's a sandbox on campus) and observing the results. Being creative and trying many shapes builds an intuition of where the ridge lines form, and what basic rules they follow.
5. An example worksheet can be found on the SUPER-M site for this activity, which can be modified to fit our needs.
6. Whatever the age, this is a very geometric activity. Remind/teach students to use proper vocabulary! Remember that 2D shapes and 3D shapes have specific names (i.e. There are no 3D triangles – there are pyramids and prisms).
7. Every granular substance has its own angle of repose. One idea for a science lab is to calculate what that angle of repose is for various substances (see the Wikipedia page for ideas of how to measure this angle). Conjecture on what affects the angle (density, granular size, mass, shape, moisture, etc.).

Handouts

Below are the handouts for the students

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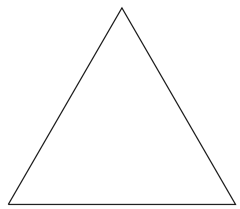
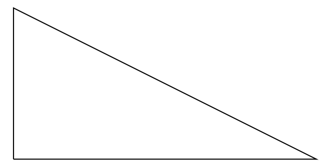
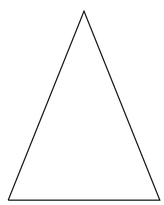
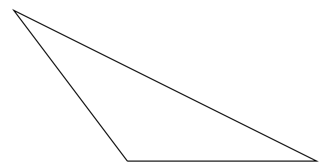
Salt Pouring Geometry: Triangles

First, cut out each of the different triangles from the index cards (or card stock)

Equilateral triangle: 

Right triangle: 

Isoceles triangle: 

Obtuse triangle: 

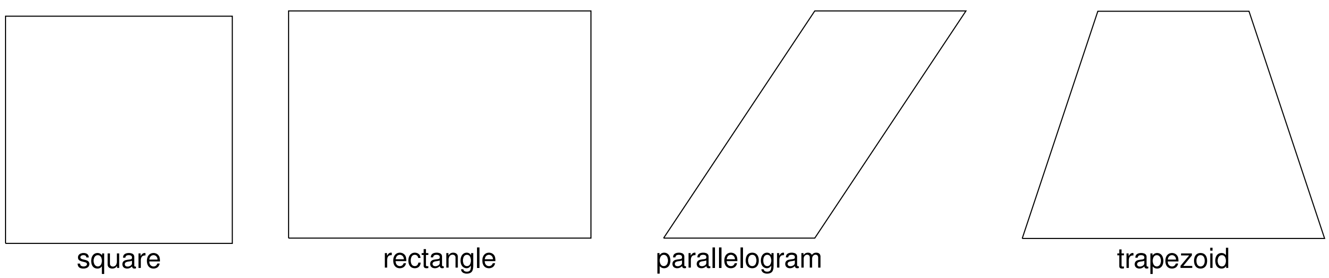
a) Set each shape over the cups in the containers and pour salt over the shape. Sketch the resulting ridge lines.

b) What do you observe about the resulting salt solids? What do you observe about their ridge lines?

c) If you pour the slat multiple times over the same shape, is the salt solid always the same?

Salt Pouring Geometry: Quadrilaterals

Now, cut out the following quadrilaterals and pour salt on top:



a) Set each shape over the cups in the containers and pour salt over the shape. Sketch the resulting ridge lines.

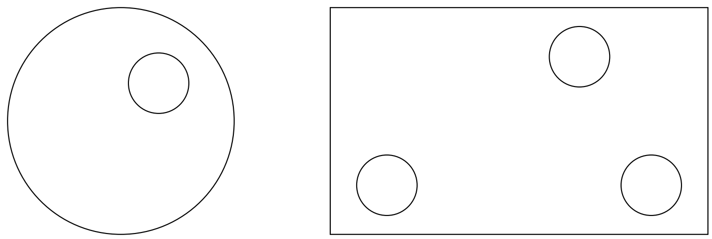
b) What do you observe about the resulting salt solids? What do you observe about their ridge lines?

c) The square produces a pyramid when salt is poured atop. Would any other quadrilateral produce a pyramid?

Salt Pouring Geometry: Circles, Holes, and Your Own

Now, cut out a circle. What is the resulting solid?

Try the following shapes with holes. What do you observe?



Now try your own shapes. Describe what “rule” the ridge lines seem to follow. Can you predict what the solid will look like before you pour the salt?