The Golden Ratio: Slide Togethers

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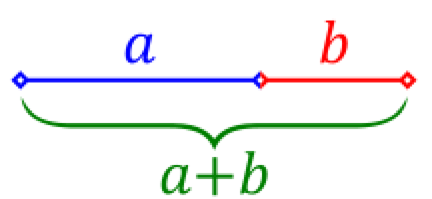
1 Introduction

The Golden Ratio has fascinated many great minds, including mathematicians, artists, architects, and biologists, throughout history. One of the earliest believed uses of the Golden Ratio was by an ancient Greek architect named Phidias around 400 BC. His statues in the Parthenon seem to be proportioned after the Golden Ratio. This is why we often use the Greek letter phi, ϕ, to represent the Golden Ratio.

A Greek mathematician Euclid was the first to record the definition of the Golden Ratio in his book *Elements* around 300 BC. The book states, "A straight line is said to have been cut in extreme and mean ratio when, as the whole line is to the greater segment, so is the greater to the less." Where extreme and mean ratio was the old name for the Golden Ratio.

Ancient Greek mathematicians became interested in the Golden Ratio because of its natural presence in regular pentagons. The diagonals of the pentagon are in the Golden Ratio to the sides of the pentagon.



$$\frac{a+b}{a} = \frac{a}{b} = \phi$$

2 Length and Objectives

The lesson is designed for a 90 minute session. Students will construct regular pentagons of the same size using compass and straightedge. Then they will use the Golden Ratio to cut slits in the pentagons in such a way that the pentagons can be put together to make a 3-D object.

3 Prerequisites

Students need a basic understanding of proportion. It is not necessary but helpful if they have basic skills with straightedge and compass constructions, e.g. bisecting lines.

4 Grade Levels and Topics

This lesson can be applied to any grade 6-12 math class. The lesson includes topics in Geometry and Algebra.

5 Common Core Standards

(Note: This lesson has only been aligned with high school standards.)

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Modeling

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods.

N-Q: Quantities

Reason quantitatively and use units to solve problems.

2. Define appropriate quantities for the purpose of descriptive modeling.

2. Choose a level of accuracy appropriate to limitations on measurement when reporting quatities.

A-SSE: Seeing structure in Expressions.

Interpret the structure of expressions.

1. Interpret expressions that represent a quantity in terms of its context.
   a. Interpret parts of an expression, such as terms, factors, and coefficients.
   b. Interpret complicated expressions by viewing one or more of their parts as a single entity.
2. Use the structure of an expression to identify ways to rewrite it.

Write expressions in equivalent forms to solve problems.

1. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

A-CED: Creating equations

Crate equations that describe numbers or relationships.

1. Create equations and inequalities in one variable and use them to solve problems.
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

A-REI: Reasoning with equations and Inequalities.

Understand solving equations as a process of reasoning and explain the reasoning.

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
2. Solve simple rational and radical equations in one variable, and give examples showing hoe extraneous solutions may arise.

Solve equations and inequalities in one variable.

4. Solve quadratic equations in one variable.

   a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions.

   b. Solve quadratic equations by inspection, taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation.

Congruence

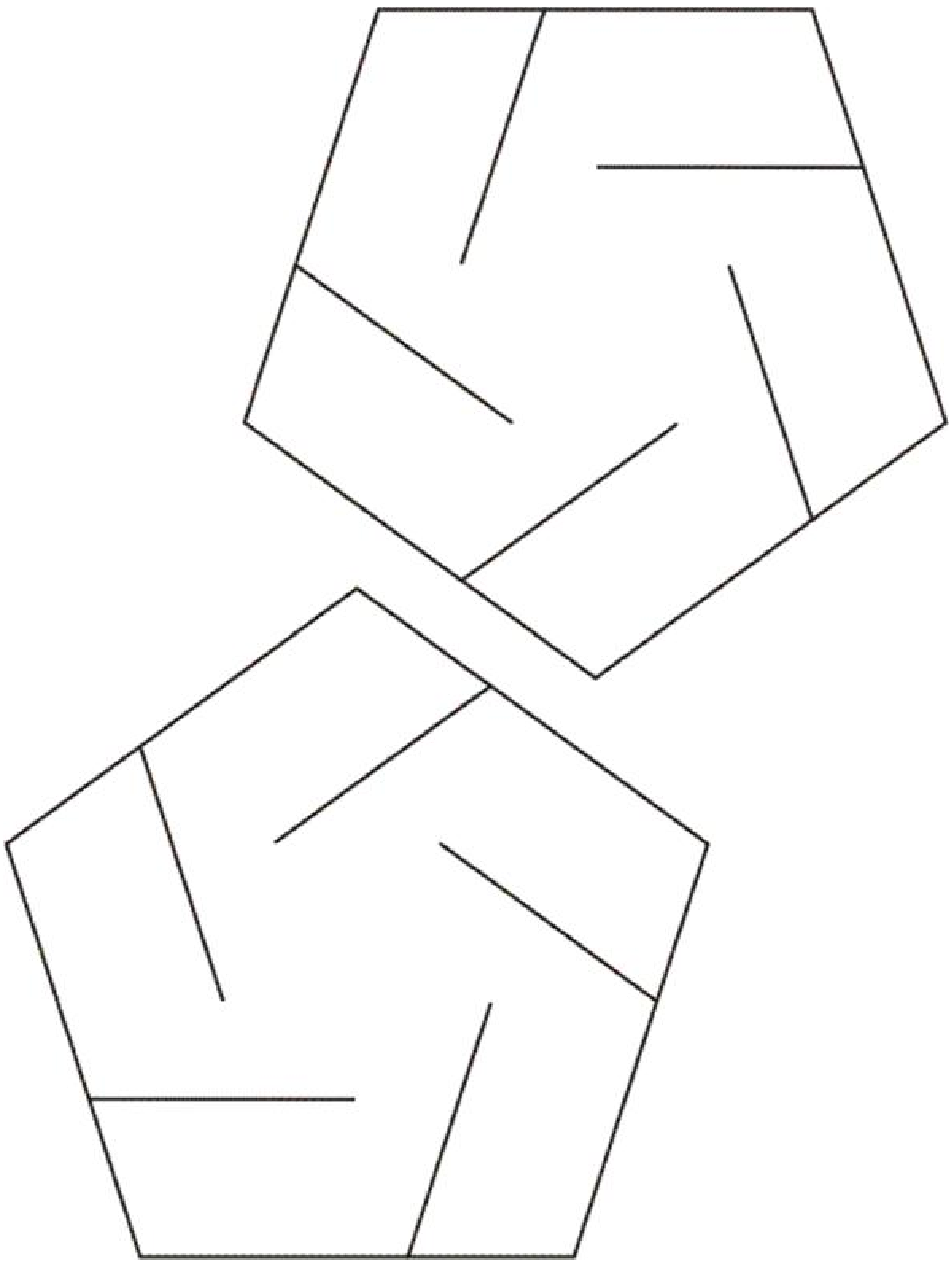
Make geometric constructions

12. Make formal geometric constructions with a variety of tools and methods.

6 Procedure

Time	Procedure
30 minutes	The lesson begins with the introduction of the Golden Ratio and a refreshment for the students on what proportions are and how to use them. The teacher may also go over simple straightedge and compass constructions necessary for constructing a regular pentagon. If the next step in the procedure is skipped, the students should be given a numerical approximation of the Golden Ratio
Optional	Students are given the relation $\frac{a+b}{a} = \frac{a}{b} = \phi$ and they obtain the quadratic equation $\phi^2 - \phi - 1 = 0$. At this point, they can graph and/or solve the equation. In the end, the students should have a numerical approximation for the Golden Ratio.
15 minutes	Students are broken up into groups. (Note that, in order for the slide togethers to fit, they must all be the same size. There are twelve pentagons per slide together. It is up to the teacher to determine how they are going to ensure that they have enough pentagons of the same size.) The teacher presents the process of constructing a pentagon. It is the groups' responsibility to keep up and help any who fall behind.
5 minutes	Once the pentagons are made there should be a discussion about where the slits should be cut and how to ensure that the slits are parallel to the sides. The teacher can explain that the slits' size and position are dependent on the Golden Ratio in regard to the pentagon's side.
5 minutes	The students will have to compute the size of the slit making sure that it is in the Golden Ratio with their own pentagon's side length. Time is given for each student to draw their slits. Then students cut out the pentagons and the slits.
35 minutes	Depending on how many pentagons are made, builders are chosen to put together the slide togethers. The construction is not straightforward, and thus, it is the groups' job to help their builders find the best way to build it.

The timeline of the lesson is highly dependent on the age and skill level of the students participating and should be adjusted accordingly. This lesson is best split between two 90 minute sessions.



7 References

http://www.georgehart.com/slide-togethers/slide-togethers.html