‘Ulu Maika

Introduction

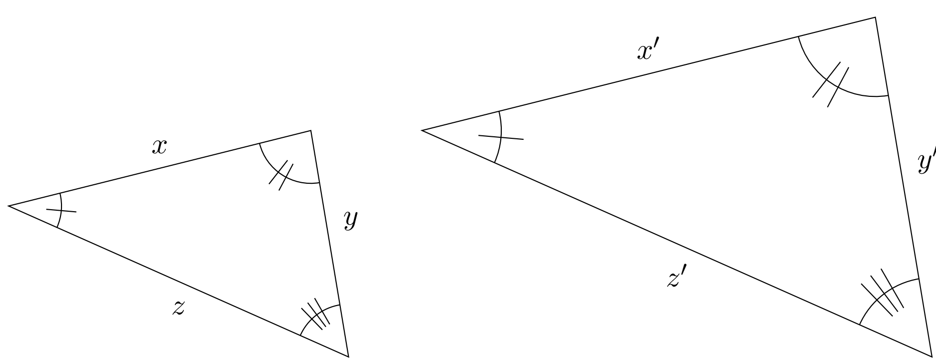
Early Hawaiians devoted large amounts of time to games, amusements, and relaxing pastimes. Games were played to develop strength, endurance, and skills. ‘Ulu maika (or ‘olohū), one of the most popular sports in early Hawai‘i, is an example a skill game where competitors roll stones towards two stakes, the victor decided upon by a variety of criteria (proximity to stakes, furthest thrown, etc.) In early Hawai‘i only men were allowed to roll the stone disks, ‘ulu, between stakes to test their skill or down long courses, free of stakes, to show their strength. Even to this day the sport is played, as hundreds of the skillfully fashioned stones of the era existing in museums and private collections. Unfortunately, many kahua maika (specially prepared courses on which the stones were rolled) used in the days of the early Hawaiians have been destroyed.

Grade Levels and Topics

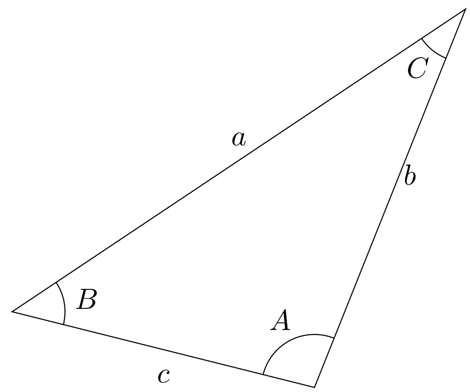
This activity is intended for high school students as a geometry activity.

Geometry:

- **Pythagorean Theorem:** $a^2 + b^2 = c^2$ where c is the hypotenuse of a right triangle with legs a, b.
- **Similar triangles:** Triangles are similar if they have the same shape, but can be different sizes. The property of two triangles that are similar is that you can multiple all the sides of one triangle by the same number to get the resulting triangles sides (as shown bellow).



- **Law of sines and law of cosines:** If we have the following triangle (next page):



Then the following equalities hold true:

– Law of sines:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

– Law of cosines:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Objectives

- Students should gain an understanding of what mathematical models are.
- Students will learn how to apply and manipulate the Pythagorean theorem, law of sine, law of cosine.
- Students should gain more familiarity with similar triangles.

Note: the images are not to scale. For example, the pegs on an ʻulu maika field are supposed to be 10 in. apart, but on the images it does not look 10 in. apart. Mathematical models/diagrams do not need to be drawn to scale; they are only used as a visual aid. In the homework problems, they will use the models to solve certain measurements about ʻulu maika rolls by applying properties of triangles.

Activity

Materials and Resources

1) Handouts and worksheets (see below)
2) Stop Watch
3) Rolling Measure (tape measure would also work, but is limited and less efficient)
4) String (used to visualize the triangle on the field)

5) Protractor (used for a more trig based lesson)

6) ʻUlu maika disk and goal sticks. Here are some examples:

NOTE: If you do not have access to ʻulu maika materials, don't worry! For the ʻulu maika, all you need is a 3in diameter cylindrical shaped item or you can make one using either commercial quick drying cement or plaster of paris. For the goal, you need two sticks.

Procedure

Geometry Part 1: Right Triangles

1. Set up the ʻulu make field prior to class
2. Organize your class into teams of 3-4 players.
3. Each team will have 3 positions:
   - 1 person to act as timer with the stop watch
   - 1 person to act as the distance recorder with the measuring tool
   - 1 person to throw the ʻulu
4. Have the students record their measurements in the tables (provided in the handouts below).
   - If there is an abundance of ʻulu resources available, each team can play their own ʻulu maika, alternating through the positions.
   - Otherwise, teams will alternate rolling and measuring the results.
   - Either way, students should record the data of the other teams/players, not just their own team or self.
5. Using this data, the students must determine a set of rules. In other words, should winner be determined by the distance of the roll, the distance from the goals, or the speed? Do the rules make sense? If the units of measure change, will the winning team change?

Example outcome table of the students' data after playing ʻulu maika.

Table for part 1: Right Triangles

Team/Player	Distance ʻulu traveled (ft)	Starting distance from goal (ft)	Time traveled (sec)	Speed of ʻulu (ft/sec)	Angle ʻulu diverged off straight path
Nā ʻŌʻō	15	2	2	7.5	10
Nā ʻAmaʻama	16	4	2	8	20
Nā Aliʻi	21	0.5	3	7	5
Nā Koa	25	8	2	12.5	25

Geometry Part 2: Law of Sines and Law of Cosines

Discussion

- In the previous section we saw one way of determining which ʻulu roll is most accurate. In this section we see new ways of determining the most accurate throw. In this version of the game, students will measure the distance they are away from the goal when they rolled the ʻulu, the distance from the center of the goal to the ʻulu, and the distance from the starting position to the ʻulu. With this information, the students will create a model of their throw. The problems will show the students how you can determine the most accurate throw using these distances.

- The law of cosines are a special case of the Pythagorean theorem. You can show them why, or have them prove why that is, using a generalized right triangle.
- Discuss with the students what happens if we use the angle θ in number 1 to compare accuracies. In this case the winner will be the one with the smallest angle. Where as in number 3 if we use the angle θ to compare accuracies then the winner will be the one with the angle closest to $180°$.

Example outcome table of the students' data after playing ʻulu maika.

Table for part 2: Law of Cosines & Law of Sines

Team/Player	Distance ʻulu traveled (ft)	Starting distance from goal (ft)	Time traveled (sec)	Speed of ʻulu (ft/sec)	Angle opposite of the ʻulu path
Nā ʻŌʻō	15	2	2	7.5	30
Nā ʻAmaʻama	16	4	2	8	40
Nā Aliʻi	21	0.5	3	7	175
Nā Koa	25	8	2	12.5	141

The following pages contain tables and worksheets for the students. Please note the second to last page is the answer key.

‘Ulu Maika

1. Organize yourselves into teams of 3-4 players.
2. Each team will have 3 positions:
   - 1 person to act as timer with the stop watch
   - 1 person to act as the distance recorder with the measuring tool
   - 1 person to throw the ‘ulu
3. Record all the measurements in the tables (provided below) for each roll of the ‘ulu.
4. Using this data, how do we determine a winner? Strength, speed, or accuracy? If the units of measure change, will the winning team change?

Table for part 1: Right Triangles

Team/Player	Distance ‘ulu traveled (ft)	Starting distance from goal	Time traveled (sec)	Speed of ‘ulu (ft/sec)	Angle ‘ulu diverged off straight path

Table for part 2: Law of Cosines & Law of Sines

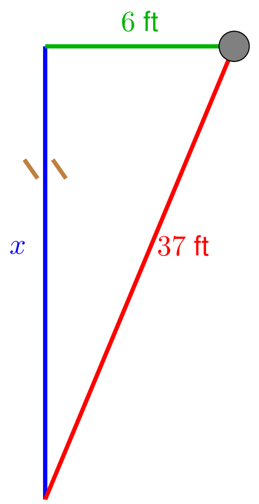
Team/Player	Distance ‘ulu traveled (ft)	Starting distance from goal (ft)	Time traveled (sec)	Speed of ‘ulu (ft/sec)	Angle opposite of the ‘ulu path

Geometry Part 1: Right Triangles

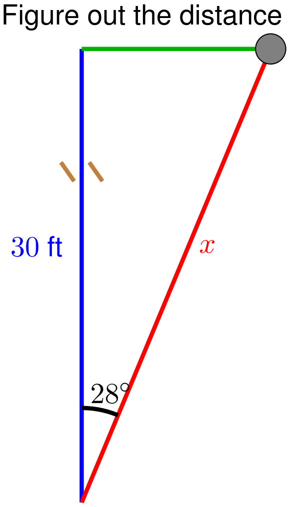
‘Ulu MaikaTriangles

1. Suppose the goal is 10 in. wide and 20 feet away. At what angle will you completely miss the goal? What is the largest angle that guarantees you will make it in the goal?

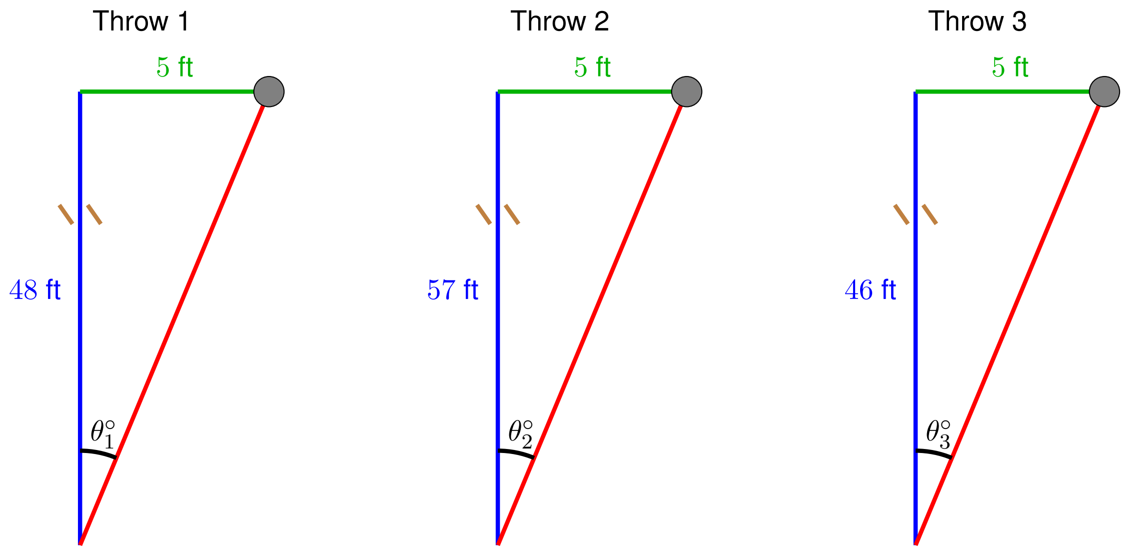
2. Figure out the vertical distance x the disk traveled.



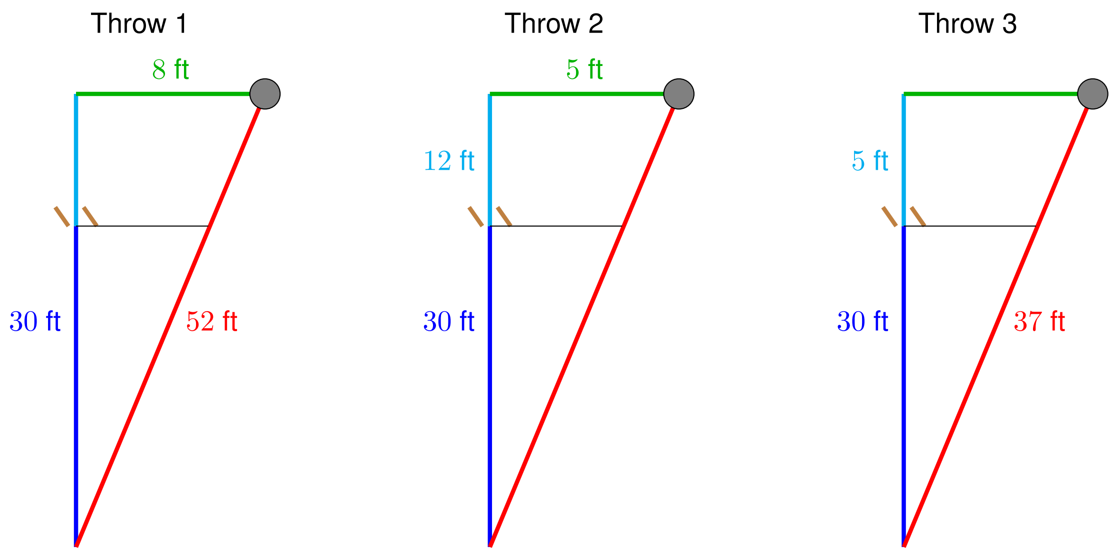
3. Figure out the distance the rock traveled x.



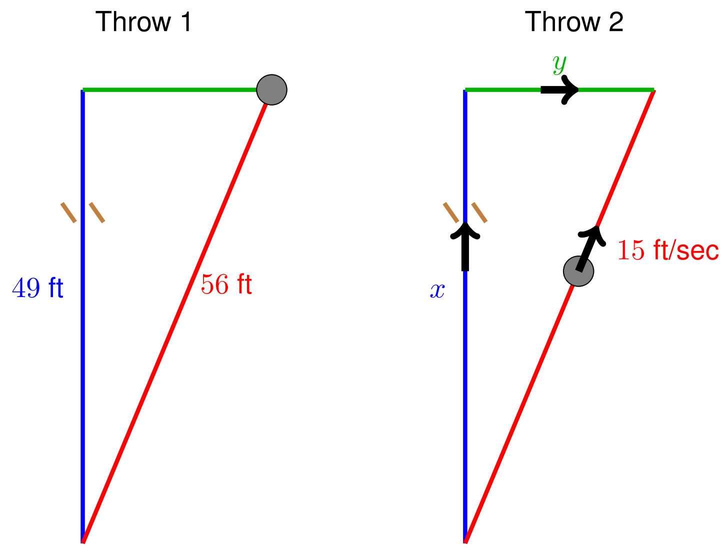
4. Among the previous problems, determine which ‘ulu roll was the most accurate by calculating and comparing the angles that the ‘ulu has diverged off of the straight path θ_1, θ_2, and θ_3. Determine which roll was the most accurate using your intuition. Now prove that your intuition is correct.



5. The person with their disk closest to the goal at the time it passes the goal is most accurate. Use this fact to determine which of the following ‘ulu rolls were the most accurate.



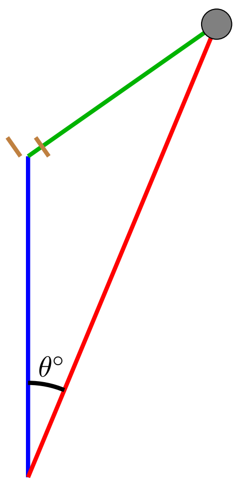
6. Use the two models (left: showing the distance the ‘ulu traveled; right: showing the velocity of that same ‘ulu) to determine the horizontal speed y and the vertical speed x of the disk roll.



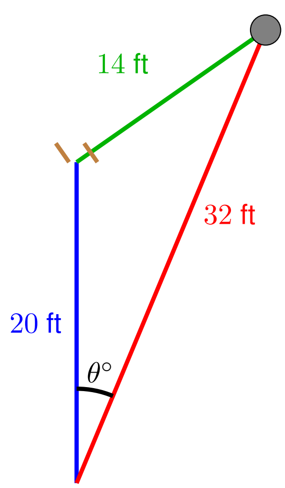
Geometry Part 2: Law of Sines and Law of Cosines

Questions

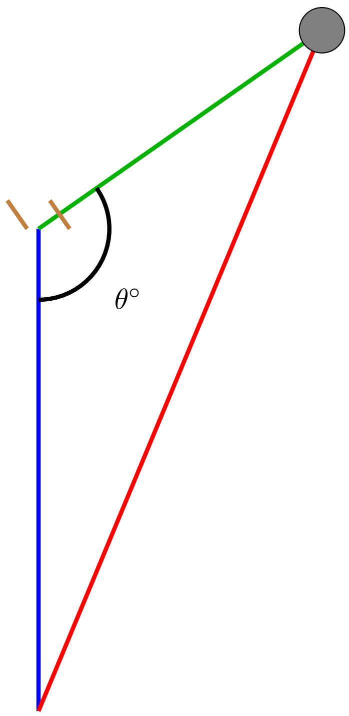
1. We are comparing multiple throws from people competing in an ‘ulu maika tournament where the winner is the person who has the most accurate throw. If we only have the angle θ from everyones’ throw, then which θ value would be the winner?



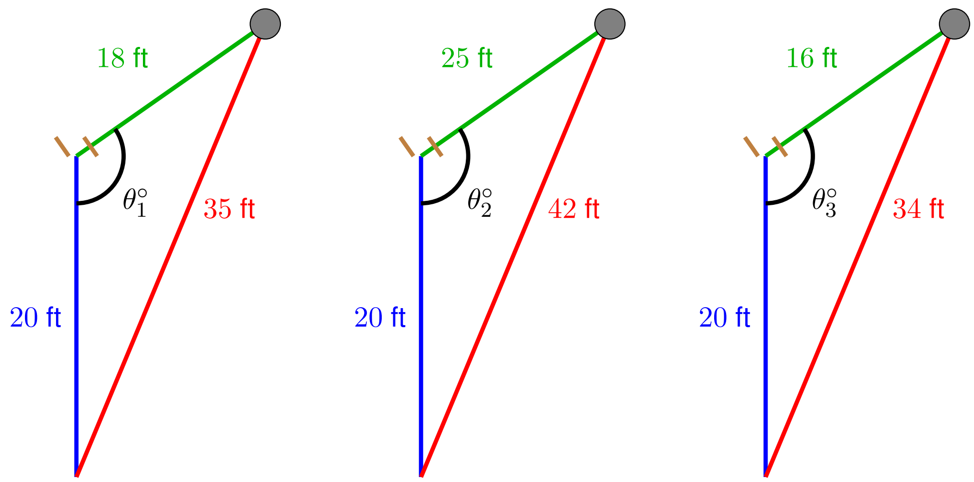
2. Determine the angle that the player’s *ulu* was off of the straight path θ using law of cosines.



3. We are comparing multiple throws of different players and we are using the angle θ, as defined in the picture below, to determine which throw was the most accurate. Which θ value would determine the winner? Hint: Think about what value of θ would be a "perfect" throw.



4. Determine which throw was the most accurate and least accurate by comparing θ_1, θ_2, and θ_3.



Answer Key

Geometry Part 1: Right Triangles

1. Rolling the 'ulu at an angle larger then $2.386°$ (or $1.193°$ to either the left or right) will result in missing the goals.
2. $x = \sqrt{37^2 - 6^2} = \sqrt{1333} \approx 36.510272523$
3. $\frac{x}{\sin(90°)} = \frac{30}{\sin(62°)} \implies x \approx 33.97710152$
4. $\theta_1 = 5.9469°$
   $\theta_2 = 5.0131°$
   $\theta_3 = 6.2034°$
   Hence throw 2 was the most accurate.
5. Throw 1: $\frac{60}{\sqrt{165}} = 4\sqrt{\frac{15}{11}} \approx 4.67099$
   Throw 2: $\frac{25}{7} \approx 3.571429$
   Throw 3: $\frac{72}{7} \approx 10.2857$
   Hence the Second roll is most accurate, roll 1 is second to most accurate, and roll 3 is least accurate.
6. $x = \frac{105}{8} = 13.125 y = \frac{15}{8}\sqrt{15} \approx 7.26184$

Geometry Part 2: Law of Sines and Law of Cosines

1. The winner would be the person with the θ value closest to zero.
2. $\theta \approx 23.7689$
3. The player with a θ closest to $180°$ would be the person with the most accurate throw.
4. $\theta_1 \approx 134.094$, $\theta_2 \approx 137.646$, and $\theta_3 \approx 141.375$. Hence roll 3 is most accurate, roll 2 is second, and roll 1 is least accurate.

Common Core Standards

- CCSS.Math.Content.HSG-SRT.B.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

- CCSS.Math.Content.HSG-SRT.C.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

- CCSS.Math.Content.HSG-SRT.C.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.(Modeling)

- CCSS.Math.Content.HSG-SRT.D.11 (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).