The School and University Partnership for Educational Renewal in Mathematics
An NSF-funded Graduate STEM Fellows in K–12 Education Project
University of Hawai'i at Manoa, Department of Mathematics

Binary Messages

Grade Levels

This activity is intended for grades $7 - 12$.

Goal

In this less, students will learn how to write numbers in binary. Numbers will then be associated with letters to write coded messages using just 0's and 1's.

Introduction

Binary digits are used all the time in computers. Binary is basically a system that is base 2, rather than base 10, which is what we are more familiar with. Normally, we change digits at every 10, but in binary, we change with every 2. Thus, the only required numbers are 0 and 1. It may seem complicated at first, but in actuality, it is not more complicated than our base 10 system – we've just become comfortable with 10s due to years of practice. Computers use binary constantly because all sorts of information can be reduced to the proper series of on-and-off switches. The following Brain Pop video is a great introduction to binary:

http://www.brainpop.com/math/numbersandoperations/binary/preview.weml.

The activities are optional.

We first want to be able to write any number from 0 to 25 as the sum of powers of 2, without using any power more than once. Powers of 2 are used because that is what it means to write in binary. Every number is written in terms of powers of 2, instead of powers of 10. For numbers between 0 and 25, we only need the following numbers: $0, 1, 2, 4, 8$, and 16. In other words, we only need $2^0, 2^1, 2^2, 2^3$, and 2^4.

Try the following examples:

1. Write 7 as a sum of powers of 2 without reusing any power.

    - **Solution:** $7 = 2^2 + 2^1 + 2^0 = 4 + 2 + 1$

2. Write 20 as a sum of powers of 2 without reusing any power.

    - **Solution:** $20 = 2^4 + 2^2 = 16 + 4$

3. Write 25 as a sum of powers of 2 without reusing any power.

    - **Solution:** $25 = 2^4 + 2^3 + 2^0 = 16 + 8 + 1$

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Any number can be written in this manner. From here, it is a simple step to write numbers in binary. We simply begin at the right side and work towards the left, writing a 0 if we do not use a power and a 1 if we do use a power. For example, $20 = 2^4 + 2^2$. Since we did not use either 2^0 or 2^1 in our sum, we begin with two zeroes from the right-hand side. Since we did not use 2^3, we use a 0 to represent this fact. Finally, we use 1 to represent the fact that we used 2^4. Thus, we would write $20 = 10100$.

To reiterate, $20 = \underline{2^4} + \underline{0} + \underline{2^2} + \underline{0} + \underline{0}$, so we write 10100 in binary. Here are the first 5 numbers in binary:

- $1 = 1$

- $2 = 10$

- $3 = 11$

- $4 = 100$

- $5 = 101$

Notice that there is a 1s place, a 2s place, a 4s place, etc., just as in standard numbers where we have a 1's place, a 10's place, a 100's place, etc.
    Try writing the following numbers in binary:

1. 10

    - **Solution:** $10 = 1010$

2. 19

    - **Solution:** $19 = 10011$

3. 50

    - **Solution:** $50 = 110010$

We now will associate a number from $0 - 25$ with each letter toward sending code messages. Copy the following table at the top of your paper:

A	B	C	D	E	F	G	H	I	J	K	L	M
0	1	2	3	4	5	6	7	8	9	10	11	12

N	O	P	Q	R	S	T	U	V	W	X	Y	Z
13	14	15	16	17	18	19	20	21	22	23	24	25

What is the maximum number of digits that we need to write any number from 0 to 25 in binary? (The answer is 5). We can send any letter we want to using 5 or less binary digits. However, if we send a series of 0's and 1's, the receiver of the message will not know where one letter cuts off from the next. What is the remedy? We send any letter using 5 spaces. Thus *A* becomes 00000 instead of just 0. Note: the scheme we devised here is very simple and unlikely to be used by an actual computer.

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Messaging

1. Try writing out our initials using just 0's and 1's. For example, *JK* would become 0100101010.

2. Try writing a one or two-word message to a partner. Decode your partner's message. When decoding, you can add commas after every 5 digits to distinguish letters more easily.

3. Now suppose that in the process of sending a message, there will always be exactly one digit that flips for every ten digits sent. How can we ensure that the receiver gets the correct message? *This sort of question frequently arises in the field of errors control coding.*