Bubble Films

Grade Levels

This activity is intended for grades $7-12$.

Objectives and Topics

The object of this lesson is for students to learn how to solve a difficult optimization problem through the utilization of scientific methodologies and to explore the applications of minimum spanning tree problems. Students will also build geometric vocabulary and develop inquisitive minds, whilst promoting scientific practices.

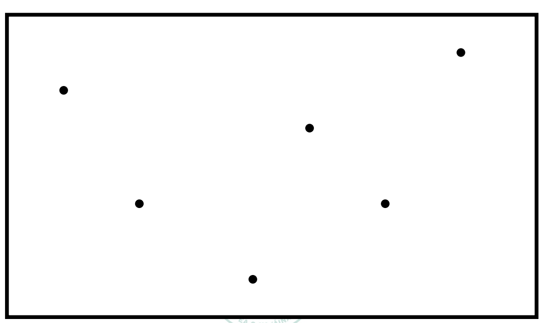
This lesson covers topics in geometry, scientific processes, optimization, energy, and engineering. The lesson will take approximately 1-2 periods, depending on the level of rigor desired.

Introduction

Some Motivating Scenarios

1. A new sub-marine data cable needs to be laid across the Pacific Ocean, connecting several major cities in Asia, North America, and South America. The longer the cable, the more it will cost, so what is the cheapest way to lay the cable?
2. A new road network needs to be built between several cities in your state. Every city must be a part of the road network. Since building roads costs money, what is the cheapest way to build the network?
3. A new electrical grid needs to be built for your town. Due to the budget, however, you need to use the existing electrical substations and power plants. Electrical cable costs money. How do you most efficiently connect the substations and power plants together?

All of the above scenarios would start with a map of "nodes" (or points), like the one below.



Each question asks how to connect the nodes with the least amount of drawing necessary. Try it!

How Do We Know?

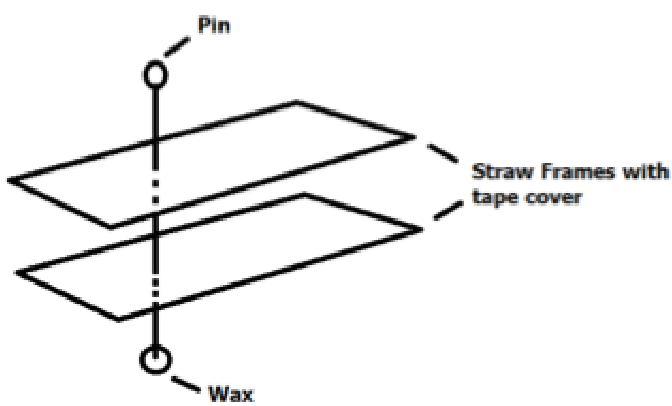
It is easy to draw a network that works. But can it be proved that a network has the shortest total length (what mathematicians would call the "minimum spanning tree")? The mathematics for this sort of problem is actually quite complex, but there is another way!

Bubbles

Soap films take the shape that minimizes surface area. This is why bubbles are always spheres – spheres are the most efficient way to enclose a constant volume. We'll use this property of soap bubbles to solve our minimum spanning tree problem.

Materials

- Bubble solution in dipping container
- Plastic Straws
- Clear transparent tape
- Pins
- Wax, clay, mounting tac, or other material to cover pin points
- Scissors



Construction

1. Bend and cut the straws to form two rectangular frames.

2. Cover each of the frames with a layer of tape (just on one side).
3. Stack the frames on top of each other and slide the pins through the tape layers in the positions of your nodes.
4. Cover the pin points with a small amount of wax so that the frames won't fall apart. The frames should slide on the pins.
5. Compress the frames and submerge in the bubble solution. It's important that the bubble solution gets into the frames and makes contact with the top and bottom. Try not to have a lot of bubble foam inside.
6. Draw the frames out of the bubble solution and carefully pull the frames apart so that you have nice bubble films between the pins.
7. The 2D minimal spanning tree can be seen by looking down through the tape and observing the lines created by the bubble films. You will probably have extra lines if there are any bubbles inside the frame. You can carefully pop these bubbles with your finger.

Making Bubble Solution

Making the bubble solution is a simple science project that may take two class periods to fully test and discuss. Bubble solution has three primary ingredients: Water, dish-washing detergent, and glycerin (available in drug stores or online). Corn syrup can also be used instead of glycerin. Many formulae can be found online, but here is a common one:

- 10 parts water
- 1 part detergent (Dawn or Joy recommended)
- $.25$ parts glycerine

You can make this at home, or you can ask your students to mix it, play with ratios, and (most importantly) tweak the formula, record their attempts and results, and report on what worked best. Formulas for bubble solution vary wildly in their ratios. What if more detergent is used? Less glycerine? If water quality is not great (hard water), distilled water is recommended – will that really make a difference from what they get from the tap? Think about the purpose of the bubbles: Do you really want big strong bubbles? Or do you want a flurry of tinier bubbles?

Teacher Notes

- It is important to use the sort of tape which is perfectly clear, not just translucent.
- If students get a lot of foam or bubbles in their frames, the films will not form properly. Proper submersion should get rid of foam and large bubbles can be popped. Multiple attempts may be necessary.
- Dipping containers just need to be deep and wide enough to submerge the frames. I use relatively small frames and recycle containers that used to hold turkey cold cuts. Larger containers are easier to work with, but require more bubble solution.

- This is a bit messy. Have paper towels on hand.
- Results should be repeatable and consistent – the bubble films are not random.
- Results aren't always intuitive. Three pins at the vertices of an equilateral triangle, four pins of a square, five pins of a regular pentagon – these you might predict. Six pins of a regular hexagon may surprise you (but not if you think through the math!).
- A sufficient amount of bubble solution can be expensive – luckily, bubble solution is very easy (and safe) to make yourself. See above.
- Bubble films have many more uses; some additional lesson ideas can be found below.

Bubble Films

This is a simple exploration for a single day. Bubble films don't just work in two-dimensions. They also reveal the most efficient way to connect 3D frames.

Materials:

- Bubble solution in dipping containers.
- Bendable wire (or Zome tools or old chemistry molecule kits)

Everything from electrical wire, to coat hangers, to fuzzy pipe cleaners, to fabric-wrapped imitation flower stems from a hobby store can be used. The flower stems work best, but what is important is that the solution will stay on the frame, that is, the frame will not fall apart, and that it is fairly easy to work with. For example, it is not suggested coat hangers be used; though they make excellent solid frames for demonstrations, players are needed to bend them.

Bend the wire into a cube. Dip the figure into the solution (the whole thing doesn't need to be submerged – dipping each face in separately should work). The resulting bubble films reveal the most efficient way to connect the cube edges in three-dimensions (and it's cool to look at).

For additional "cool" factors, if you can get an extra bubble into the center of the cube's bubble films (*this happens occasionally just by accident if you have bubbles floating around on the surface of your solution*), you can stick a straw into the bubble and expand it, producing a cubic bubble.

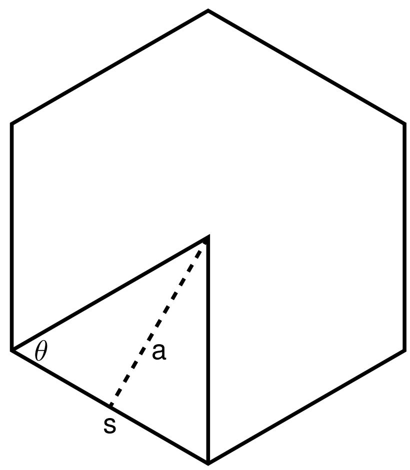
Other 3D figures can be experimented with to see what interesting bubble films can be formed. Are they intuitive? What 2D shapes seem to be most often formed by the films? If you propose a 2D shape, can the students make a 3D frame with a bubble film that features that shape? As a geometric study, make sure students use proper vocabulary.

Why would this be important? Consider manufacturing: If you produce products, you want to do so with the least amount of wasted materials. In other words, you want the most efficient process possible (because materials cost money). Bubble films can give us a clue as to how to produce the most efficient (or lightest) structures for our products, whether we're involved in architecture and construction, rocket engineering, toy design, or automotive design.

Optimal Containment

A lesson for geometry classes, or a homework/extra credit problem. I mentioned that spheres are the most efficient way to contain a given volume. Similarly, circles are the most efficient way to contain a given area (that is, it has minimal perimeter). Unfortunately, these results require calculus to prove. BUT, if we know a bit of geometry, we can conjecture about the circle.

We'll consider just regular polygons. Fix the area – let's make it 10 square units. Now, we'll calculate the perimeters of regular polygons of increasing number of sides. Students may figure this out themselves, but I'll go through one method that utilizes many basic geometric formulas:



We know the area formula for a regular polygon: $A = \frac{1}{2}aP$, where a is the apothem and P is the perimeter.
A regular n-gon has a side-length, s, of $s = \frac{P}{n}$, so $P = ns$.
An n-gon also has interiors angles that sum to $(n-2) \cdot 180$. So each interior angle has measure: $\frac{(n-2)\cdot 180}{n}$.

So, as in the figure above, we know that $\theta = \frac{(n-2)\cdot 180}{2n}$. And $a = \frac{1}{2}s\tan(\theta)$.
It follows, by many substitutions, that:

$$A = 10 = \frac{1}{2}\left[\frac{1}{2}s\tan\left(\frac{(n-2)\cdot 180}{2n}\right)\right]ns$$

And therefore:

$$s^2 = \frac{40}{n \cdot \tan\left(\frac{(n-2)\cdot 180}{2n}\right)}$$

So we can solve for s. And we know how to relate side-length to perimeter: $P = ns$.

Starting at $n = 3$ and building up with larger and larger n, you will notice that the perimeter shrinks (see note below). We therefore conjecture that the larger the n, the smaller the perimeter. We note also that the larger the n, the more a regular n-gon resembles a circle – which is sort of an infinite-sided figure. So it makes sense that a circle best encloses a given area (with perimeter $2\pi r$).

Please note: Because the numbers become lengthy decimals and start getting close to each other as n increases, rounding may cause a break in the pattern. When using calculators, students should be encouraged to keep as many decimal places as possible instead of rounding.

Further note: This is **NOT** a formal proof, but rather, it is an observation that leads to a conjecture.