Game Theory

Grade Levels

This activity is intended for students grades $6-8$.

Objectives and Topics

Game Theory is a set of activities split across six days. The purpose is to introduce students to the concepts of game theory through mathematical techniques they already know! Below is a very brief description of each day's activity.

Below you will find the outlines of each day's activity (or click the day you're interested above to skip to that day's outline).

Day 1: Practice with Fractions/Percentages

	Activity	Suggested Time	Materials	Preparation/Summary
Day 1	Gak	60 minutes	Borax, 2 trash bag per table, plastic/measuring cups, 3 cups glue per table, water, stirrers, 1 Ziploc per student, food coloring	Cover tables with trash bags, set materials on tables. Put recipe up on document projector (Elmo).
	Viscosity and Fractions mini assessment	20 minutes	Worksheet	
	First Game	20 minutes	One small piece of paper per student, one ballot box	
	Journal Making, Drawing	20 minutes	One journal per student, colored markers & pencils	Draw a mathematician.

Gak Recipe

Gak recipe (Tell the students they must make half this recipe. This is a good opportunity to talk about multiplying fraction.)

- 1 cup glue
- $1\frac{1}{4}$ cup water
- 1+ teaspoon Borax

Empty the glue into a Zlploc bag. Add $\frac{3}{4}$ cup water. Pour the glue-water mixture into the bad and mix well. Add the glue-water mixture to the glue in the mixing bowl. Put in a few drops of food coloring. Don't waste it!

Measure $\frac{1}{2}$ cup of warm water into a plastic cup and add a teaspoon of Borax powder to the water. Stir the solution – don't worry if all the powder dissolves.

While kneading the glue in the bad, slowly add a little of the Borax solution. Immediately, you'll feel the long strands of molecules starting to connect. Use your hands to do some serious mixing. Keep adding the Borax solution to the glue mixture (don't stop mixing) until you get a perfect batch of Elmer's slime. You might like your slime more stringy while others like firm slime. Hey, you're the head slime mixologist – do it your way!

When you're finished playing with your Elmer's slime, seal it up in the zipper-lock bag for safe keeping. Afterwards, this might be the right time to use the fraction mini-assessment below.

Food for thought from the Internet: http://www.paintcenter.org/rj/feb05u.cfm

Discussion

Viscosity is a measure of the **resistance** of a **fluid** which is being deformed by either **shear stress** or **tensile stress**. In everyday terms (and for fluids only), viscosity is "thickness".

Q. What is the relationship between *Density* and *Viscosity*?
A. There is no obvious relationship between density and viscosity. At the extreme, mercury has a VERY high density, 13.6 g/cc at room temperature, but its viscosity is very low. This metallic liquid can easily be poured. Water, which also pours easily has a density of only 1 g/cc. Honey (syrup) that we pour over our pancakes has a high viscosity at room temperature, but when we heat it in a pot of hot water, its viscosity drops so dramatically that we can pour it quite easily. Yet its density remains the same.

First Game

Each student has one decision to make. Whether to push an imaginary button. If he does not push the button, nothing happens. If he pushes the button, a portion of every other students Gak is taken away, but the portion taken away from him is halved. Each student has 20 little pieces of Gak. If no one pushes the button, everyone gets all their Gak to take home. If 7 out of 20 students push the button, each student loses 14 pieces of Gak, except the people who pushed the button, who only lose 7 pieces. A secret ballot is taken to let students choose whether to push or not to push.

Making a Journal Booklet

1. Have the students make their own journal booklet as you give step-by-step instructions.
2. Assign the following reflection prompt: "Reflection $\#1$: A mathematician is..." Have them draw a mathematician.
3. Allow them to decorate the cover.

Fractions Mini-Assessment

Make the following recipe (Austin's Famous Chili Grease Blast Bread) one third as large.

Eg. 1 pound cheese *changes to* _____

3 cans beans *changes to* _____

$\frac{1}{2}$ cup salt *changes to* _____

6 potatoes *changes to* _____

4 chiles *changes to* _____

$\frac{2}{3}$ tbsp. oil *changes to* _____

1.5 kg. flour *changes to* _____

Intro to Game Theory

"Game theory is the study of strategic, interactive decision making among rational individuals." It has applications to business, economics, military strategy, politics, biology, soccer, personal applications like getting your kids to do their homework, and much more. – Scott P. Stevens

Day 2: Airplanes and Brains! (Practice with Graphing and Area)

	Activity	Suggested Time	Materials	Preparation/Summary
Day 2	Reflection on Previous Day	5 minutes	Post-it notes	Have students write a reflection on yesterday, post notes at from of room
	Make 2 airplanes	40 minutes	2 sheets paper per student, preferably colored	Teach students two paper airplane designs with different surface areas. A good website for templates is: funpaperairplanes.com
	Surface Area Activity	45 minutes	Classroom whiteboard and marker, rulers and calculators (at least one per group of 4).	Review areas of rectangles, triangles, and trapezoids
	Airplane Testing	40 minutes	Tape measure, long hallway or outdoor area without wind	
	Intro to Prisoner's Dilemma	20 minutes	The students' airplanes	A game in pairs risking their planes
	Journal	15 minutes	Journals, pens. *Encourage drawings and use of color.*	In the game of life, what gives you the biggest payoff?

Reflection on Yesterday

When students are ready to begin class, have them jot down on a post-it note something about the activities of the previous session. It can be about something they learned, something they want to learn more about, something they liked or did not like, or all of these. Have students post notes on a chart paper. Address the reflections as you tell the students what activities you have planned for today.

Airplane Making

When teaching the students how to make paper airplanes or origami, address the concepts of symmetry and transformations. For example, a fold of paper can be thought of as reflecting a point across the fold line. Use

words like reflection and rotation and describe the geometric shapes appearing with the folds, including the type of shape and the angle measures of its corners.

Surface Area Activity

Assign the students the task of calculating the surface areas of their planes. We used a plane that was triangular and one that looked like a trapezoid with an adjacent rectangle. The students worked in groups of 4 to discuss how to measure the areas, and jotted down measurements and used calculators to get an accurate answer.

Plane Testing

The students can devise their own way to test planes, or you can tell them to do 5 throws and calculate the average throw distance. Have them make a hypothesis about possible relationships between surface area and flight distance. Mention other factors like throw power. As the students collect their data, have them plot it on a distance vs. surface area graph. Then address the hypothesis. The focus is more on plotting points than using the scientific method, and in practice the relationship between surface area and distance is not clear.

Prisoner's Dilemma

This game is a two by two (2 players with 2 strategies each) simultaneous game that is fundamental to game theory. The classic scenario is two criminals who have committed a crime together are being questioned by the police and must decide whether to betray (confess) or be quiet. The matrix below represents the payoffs. The first number is Prisoner A's payoff and the second is for Prisoner B. The best payoff is 3 and the worst is 0.

	Prisoner A betrays	Prisoner A is quiet
Prisoner B betrays	$1, 1$	$0, 3$
Prisoner B is quiet	$3, 0$	$2, 2$

The idea is that if both prisoner's are quiet, they get the highest combined payoff, but the dominant strategy is to betray. This models many situations in business and politics. The Cold War was a sort of prisoner's dilemma, with launching nukes symbolized by betrayal. Have the students play this game with a partner, risking the loss of the airplanes they made. Without talking to their partner, they each must decide whether to betray or be quiet. A secret ballot is taken, and the payoffs given are the number of planes the student gets to keep. If both students are quiet, they keep their planes. If they both betray, they must give the teacher a plane each. If one student is quiet and the other betrays, the betrayer gets to take the planes of the other student!

Payoff – the value of an outcome of a game

Day 3: 2 by 2 Simultaneous Games

	Activity	Suggested Time	Materials	Preparation/Summary
Day 3	Reflection on Previous Day	15 minutes	Post-it notes	Have students write a reflection on yesterday, post notes at from of room
	Origami	30 minutes	Origami paper or plain paper with scissors to cut into squares	
	Games	45 minutes	Worksheet	Play a game involving probability and get students to think about strategy
	Prisoner's Dilemma Tournament	60 minutes	Monopoly money, playing cards (one deck is enough for 30 students)	Gambling warning
	Journal	15 minutes	Journals, pens. *Encourage drawings and use of color.*	What is your strategy in the game of life to achieve your biggest payoff?

Reflection on Yesterday

When students are ready to begin class, have them jot down on a post-it note something about the activities of the previous class. It can be about something they learned, something they want to learn more about, something they liked or did not like, or all of these. Have students post notes on a chart paper. Address the reflections as you tell the students what activities you have planned for today.

Origami

A good website for origami ideas is:

http://en.origami-club.com//index.html.

We made a balloon with wings. Even more than with paper airplanes, ideas of symmetry and geometric transformations play into the lessons. The origami will be used later as payoff in the Prisoner's Dilemma Tournament.

Games see supplemental worksheet)

There are three other important two by two games on the worksheet. Let the students spend some time discussing in groups the appropriate payoffs. As a challenge question, ask students to invent another game not covered in the class so far. Or ask how many different games there are. I.e., how many different 2 by 2 payoff matrices up to isomorphism? Isomorphism is the word of the day.

In these games, there are four possible payoffs, numbers 0 to 3 for each player. 3 is best, 0 is worst.

The Prisoner's Dilemma

Prisoner Alf and Prisoner Bob stole malasadas together. Each decides whether to confess and turn the other one in. The payoff matrix is:

	Alf be quiet	Alf betray
Bob be quiet	Alf 2, Bob 2	Alf 3, Bob 0
Bob betray	Alf 0, Bob 3	Alf 1, Bob 1

Chicken

Arch nemesis Truckers Chuck and Dude are driving straight at each other, and have to make a decision at the same instant. For Trucker Chuck, the best thing that can happen is he does not swerve and Trucker Dude does swerve. The next best thing is they both swerve. The second worst thing for Chuck is if Trucker Dude stays straight and Trucker Chuck swerves. The worst thing is if neither swerves and they crash. Fill in the payoff matrix.

	Chuck Stays Straight	Chuck Swerves
Dude Stays Straight		
Dude Swerves		

Battle of the Sexes

Mario and Princess Peach are going on a dinner date. They have to choose whether to dress fancy or casual, but their phones are broken and they can't coordinate. They both want to match, but Mario hopes its casual and Princess Peach hopes its fancy. Fill in the payoff matrix.

	Mario Goes Fancy	Mario Goes Casual
Princess Goes Fancy		
Princess Goes Casual		

Prisoner's Dilemma Tournament

An iterated Prisoner's Dilemma Tournament using computer programmable strategies was hosted by Robert Axelrod, described in his book The Evolution of Cooperation (1984). The participants submitted strategies and were played against several other participants in a round robin style, with payoffs accumulated over the whole tournament. Here a few example strategies:

1. Always be nice (be quiet)

2. Always be mean (betray)
3. Grim trigger – be nice until betrayed, then never be nice again
4. Tit-for-tat – be nice the first time, then always do whatever the other player did to you last time

Some of the strategies in Axelrod's tournaments were much more complicated. Encourage the students to guess the best strategy and choose one for their own. However, they are allowed to improvise or change their strategy as the tournament goes on. The payoffs are 0 through 3 as in the matrix given on Day 2. Put the students in groups of 4, and have each compete against the other 3 group members, with 10 iterations. This makes three rounds of ten consecutive games. Players proceed like the game Rock Paper Scissors, but there are only two choices. Rock is betray and Paper is be nice. Players who are nice to each all ten times will get a net score of 20 each, while players who betray each other every time will net 10 points. A player who is nice every time and always gets betrayed will score no points while the betrayer will score 30 points in ten games. After the tournament the overall winner gets a prize as well as each group winner. The prizes can be the origami made at the beginning of class. Axelrod held two tournaments, and Tit-for-tat amazingly won both. Have the students reflect on the winner's strategy. Cooperation tends to overcome.

Isomorphism – a correspondence between two mathematical objects that shows they are basically the same (roots iso-same, equal morph-shape)

Day 4: Probability, Expected Value, and Strategy

	Activity	Suggested Time	Materials	Preparation/Summary
Day 4	Reflection on Previous Day	15 minutes	Post-it notes	Have students write a reflection on yesterday, post notes at from of room
	Skunk	75 minutes	2 dice, a paper and pencil for each student	Play a game involving probability and get students to think about strategy
	Tiny poker	60 minutes	Monopoly money, playing cards (one deck is enough for 30 students)	Gambling warning
	Journal	15 minutes	Journals, pens. *Encourage drawings and use of color.*	What is a situation in life when you bluff?

Reflection on Yesterday

When students are ready to begin class, have them jot down on a post-it note something about the activities of the previous class. It can be about something they learned, something they want to learn more about, something they liked or did not like, or all of these. Have students post notes on a chart paper. Address the reflections as you tell the students what activities you have planned for today.

SKUNK

The game goes as follows: Every player begins standing up. After each roll of the dice, each student must decide whether to sit or remain standing. The students rack up points on each roll of the dice. If one sits she fixes her score for that round and scores no more points. The round ends when a 1 is rolled with either dice. If a student is standing when a 1 is rolled, he loses all his points for that round and gets a 0. Five rounds are played. Optional rule: If double 1s are rolled (snake eyes), every standing player loses her points for that round AND every previous round.

Many activities can be tied to this game. A discussion of probability is in order including the probability of certain rolls of the dice, like at least one 1. The probability of this is $11/36$. (Why?) Thus the probability that a standing student will get points is $25/36$. The average roll that does not include a 1 is 8 points, so the expected value for one roll is $(25/36) \times 8 = 5.555...$ However, one must include previous points in the expected payoff for standing up. The expected payoff for sitting is precisely the score already earned. By comparing these two numbers students can see the best strategy.

Gambling Warning

Write the quote "Lottery: a tax on people who are bad at math". Do an analysis of the expected value of a lottery ticket. For example, the expected ticket value for a lottery in which the player wins $\$1,000,000$ for correctly

guessing 6 numbers, $1-40$, in order, is $1,000,000/102,400,000 < 0.01$, and a ticket price of 1 cent is not worth it.

Let students know that gambling can be an addictive behavior that hurts people, and is banned in schools and many states. However, if students learn the math behind gambling they are less at risk to lose money as addictive gamblers.

Tiny Poker

Give students monopoly money or let them use paper to keep mock accounts. Tiny poker is played between two players, with only 3 cards. Two cards are identically low and one is high, for example, two sevens and an eight. Player 1 puts in $\$40$ ante and Player 2 puts in $\$70$. The players are dealt one card each. Player 1 must decide whether to fold or put in $\$70$ more. Player 2 must then decide whether to fold or put in $\$30$ more (not quite matching player 1). Students can make a game tree of strategies based on the three different possible deals-Player 1 gets the high card, Player 2 gets it, or no one gets it. If a player has a high card they should never fold. The real strategy comes in when a player gets a low card. Sometimes money can be made by bluffing. If a player never bluffs, they always lose so it is a good strategy to bluff sometimes. The percentage of the time one should bluff depends on the expected value of each situation, and the game theoretic analysis is interesting.

Example: Player 1 Gets Low Card Game Tree

This is easier done on a chalkboard than on Microsoft Word.

Two branches
Player 1 folds, payoff is $-\$40$.
Player 1 bets, go to next two branches

Player 2 folds, payoff is $\$70$
Player 2 bets, go to next two branches (show cards)

Player 2 has low card, payoff is $-\$5$
Player 2 has high card, payoff is $-\$110$

As Player 2 also decides whether to bluff when she has a low card, the analysis is similar to the soccer example. (See supplemental worksheet) Player 1 should bluff at a rate that minimizes Player 2s expected payoff and vice versa.

Probability – a number from 0 to 1 indicating the likelihood of an event.

Expected Value Worksheet

Expected value – the sum of the possible payoffs, each multiplied by its probability

Suppose we are in the S round of SKUNK. The first roll was a 12. There are two strategies at this point, sit or stand. If you sit, you get a guaranteed 12. If you stand, you risk losing it, but have a chance at gaining more. This chance is about 69%, the probability of not rolling a 1. The average amount you stand to gain is the average of all rolls that don't have a 1. This is 8. So the expected value for standing on more turn is

$$(payoff) \times (probability) = (12 + 8) \times 0.69 = 13.8$$

Since the expected value for standing is greater than the 12 points for sitting, it is worth it to stand.

Suppose you stand, and luckily, a 10 is rolled. You now have 22 points guaranteed if you sit. But you might want to stay standing. What is the expected value for standing?

$$(payoff) \times (probability) = \_\_\_\_\_ 0.69 = \_\_\_\_\_$$

Is it smarter to stand or sit? _____

Practice

1. You play a game in which your friend flips a quarter, and if you call it, you get it. What is the expected value of your payoff?
2. A casino shark offers you to play a game of chance. You roll a three sided die (!?) that has a 50% chance of showing a 1, 30% chance of showing a 2, and 20% chance of showing a 3. A 1 pays you $\$100$, a 2 pays you $\$200$, and a 3 loses you $\$300$. Is it worth playing?
3. In a game of poker, 7 card no-draw, you have two aces. Suppose you can only be beaten by three of a kind. You have $\$50$ in the pot, and you just got raised $\$50$. Do you call?

Soccer penalty kick analysis (from Scott P. Stevens *Games People Play*)

Successes

	Goalie Left	Goalie Right	
Kick Left	132	323	455
Kick Right	347	333	680
	479	656	1135

Attempts

	Goalie Left	Goalie Right	
Kick Left	227	340	566
Kick Right	373	477	850
	599	817	1416

Expected value of Success for kick

	Goalie Left	Goalie Right
Kick Left	0.58	0.95
Kick Right	0.93	0.70

We see that when the kicker kicks left, he scores 58% of the time when the goalie goes left, and 93% of the time when the goalie goes right (guesses wrong). If the goalie goes left with probability q, and right with probability $1 - q$, the expected value for the kicker kicking left is:

$$0.58q + 0.93(1 - q)$$

Doing the same analysis for the kicker kicking right, the expected value is:

$$0.95q + 0.70(1 - q)$$

Set these expressions equal to each other and solve to get the best q for the goalie.

Day 5: Social Study/Technology

	Activity	Suggested Time	Materials	Preparation/Summary
Day 5	Reflection on Previous Day	15 minutes	Post-it notes	Have students write a reflection on yesterday, post notes at from of room
	Social Experiment	15 minutes	Index cards, TI-Inspires	Set up the calculators to be able to graph data communicated with other calculators. Write on index cards (or make it part of students' task). See below.
	Chart Making	60 minutes	Large graph paper. 1 per group of 4 students	Goal is to visualize the data from the experiment
	Journal	15 minutes	Journals, pens. *Encourage drawings and use of color.*	When has moral hazard (cheating because you won't get busted) influenced your life?

Reflection on Yesterday

When students are ready to begin class, have them jot down on a post-it note something about the activities of the previous class. It can be about something they learned, something they want to learn more about, something they liked or did not like, or all of these. Have students post notes on a chart paper. Address the reflections as you tell the students what activities you have planned for today.

Social Experiment

Game theory has been used to make startlingly accurate predictions in human behavior from politics to soccer penalty kicking. This demonstrates a game in which the mathematical model habitually fails to match human behavior. The game is called the Ultimatum game, and is quite simple. 2 players have the opportunity to win a combined total of say $\$10$. Player 1 decides how to split the money, and player 2 chooses either to accept the split or not. If Player 2 does not accept, both players get nothing. If money is the only payoff, the clear strategy for Player 1 is to offer a split of $\$9.99$ for herself and 1 cent for Player 2. And Player 2 will accept, since 1 cent is better than none. It is not hard to guess that Player 2 will usually reject this offer. Why?

Game theorists and behavioral scientists have studied this in depth and used research money to actually play the game hundreds of times with test subjects. The results are often at odds with a mathematically reasoned prediction. The most common offer is a $50/50$ split, and many players have turned down offers between $\$20$ and $\$40$ out of $\$100$. Why would someone throw away $\$40$ of free money? In the classroom we repeat the experiment, with modifications that may or may not be helpful. One modification is the lack of real money; we have to ask the students to pretend. Another is the method of offering. We wish to collect data on the frequency with which various offers are accepted. To do this, we use index cards to denote different offers. For example, an index card might read, "I get $\$60$, you get $\$40$" or "I get $\$99.99$, you get $\$0.01$". We want one card per

student, and we might like the offers to be evenly distributed. The students conduct the experiment by going around to every other student in the class and making the offer that is on the card. The student being offered decides whether he would accept or reject that offer, and the offerer records how many acceptances/rejections she gets. The students are now given the task of visualizing their data with the TI-Nspire. We had a feature that enables students in groups of 4 to enter data from their own calculator and have a bar graph made using all 4 data pieces from the group. The students should discuss reasons for the shapes of their graphs.

Chart Making

While the calculators work for groups of 4, we needed to compare the data of the whole class. Unfortunately we could not do this on the calculators in the same manner, so we had the students devise a table to record all the data from the class. Then we had a group competition to see who could make the best chart that clearly showed all relevant features of the data. In discussing the final charts, some odd phenomena were apparent. Some students got more rejections for better offers! It was apparent that the way in which the offer was made, and not just the value, was an important factor in acceptance. And we compared our data to some that had been done in the actual experiment. (According to Scott Stevens lectures on Game Theory by the Teaching Company). Our offers were more commonly accepted than in the real experiment. One reason could be that the students were not allowed to choose their own offer, since they just offered what was on their card. In light of this, we might suggest modifying our experiment to include a more accurate playing of the Ultimatum Game. And this relates to the word of the day, which will be explored more tomorrow.

Moral Hazard – The problem, especially in business and politics, arises when decision makers are not directly responsible for the consequences of their decisions.

Day 6: Pascal's Triangle

	Activity	Suggested Time	Materials	Preparation/Summary
Day 6	Reflection on Previous Day	15 minutes	Post-it notes	Have students write a reflection on yesterday, post notes at from of room
	Cash in a Hat Game	30 minutes	A hat, play money	A game demonstrating moral hazard
	Making the triangle	45 minutes	Chart size graph paper, 1 sheet / group of 4	Find the pattern in Pascal's triangle
	Combinatorics	60 minutes	Youtube video on the Universal Counting Principle (optional), various pieces of colored cloth or papers, balls	Play with combinations
	Journal	15 minutes	Journals, pens. *Encourage drawings and use of color.*	How many possible outfits do you have to wear? Include crazy ones!

Reflection on Yesterday

When students are ready to begin class, have them jot down on a post-it note something about the activities of the previous class. It can be about something they learned, something they want to learn more about, something they liked or did not like, or all of these. Have students post notes on a chart paper. Address the reflections as you tell the students what activities you have planned for today.

Cash in a Hat

This game has 2 players, and can be demonstrated first with a volunteer from the class. Player 1 must put either \$0, \$1, or \$3 in a hat. Player 2 must either match the cash in the hat, or take the cash out of the hat to keep, with Player 1 losing it. If Player 2 matches the cash, both players get their money back, and an additional amount as follows:
Matches \$1

1. Player 1 gets an additional \$1
2. Player 2 gets an additional \$1.50

Matches \$3

1. Player 1 gets an additional \$3
2. Player 2 gets an additional \$2

A game tree will show that if Player 1 puts in $\$1$, the highest payoff for Player 2 is to match the cash and get $\$1.50$. But if Player 1 puts in $\$3$, Player 2 is better off taking the money. So the strategy for Player 1 will be to put in $\$1$, even though both players would make more money if only Player 2 was trustworthy. This models an investment situation in which it might be tempting for the person receiving the investment to take the money and run. This is moral hazard. Have the students play the game a while before analyzing it with them.

Making the Triangle and Combinatorics

We shift gears a little to examine the math behind some probability issues encountered thus far, and see one of the most famous mathematical patterns, Pascal's triangle. Turn your graph paper horizontally, and start by writing a 1 at the very top center square. Then move down and left a square, making a 1, and reflecting that 1 about the center line. You have three 1s now. Give the students the next two rows of Pascal's triangle, and then have them try to figure out the rest. Perhaps students will think the fifth row is 14441, but somebody might guess 14641. Give the answer as a combinatorics problem, like the middle number is the number of ways to choose 2 things out of 4. Have the students fill in the paper as much as they can and color the squares with even numbers one color and the odds a contrasting one. Here you might talk about Sierpinski's triangle and fractals. There is a huge amount of math related to Pascal's triangle, and more than one day could be spent. But our main goal is for the students to read the binomial coefficients (the number of ways to choose k things from n) from the triangle, and ponder why the relationship holds. Ask the students to find the number of ways to choose n things from 5, $n = 0, 1, 2, 3, 4, 5$. Hopefully they see the relationship. If you have colored balls or manipulatives of a similar nature, have them demonstrate the possible choices out of some small number. And if time permits have them try to figure out the number of 5 card hands chosen from a deck of 52. Another aspect of combinatorics we saw was the Universal Counting Principle. We viewed the Square One video on Youtube (a quick search will reveal many such videos) involving making a band out of various numbers of different instruments. Have the students count the number of different outfits (and wear them!) that can be made with cloth belts and bandanas of different colors. Count the number of different rolls with 2 dice (not just their values). Our goal was to explore these things with the students, rather than teach a rigid combinatorics lesson. And now our final word of the day:

Combinatorics – the mathematics of counting! (Not as easy as it may sound)