Greedy Pig Game

Grade Level and Topics

This activity is appropriate for students in grades 6-12. Middle school students are able to play the game, and discuss basic probabilities, as well as independent and dependent events. High school students can continue with a discussion on expected value.

Introduction

Dependent and independent probabilistic events can be a challenging and confusing concept. This activity is meant to get students thinking about what it really means to be an independent or dependent event. Two events are *independent* if the occurrence of one does not affect the probability of the other. Rolling die, flipping coins, and drawing cards with replacement are all independent events.

Materials

- One die
- Paper and pencil for each student

Activity

Playing the Game

- A game consists of 5 rounds
- Each round consists of a series of rolls. The teacher remains in the front of the room, and is in control of the die. Students all stand besides their desks.
- For each roll, if a 2,3,4,5, or 6 is tossed, players write down the number tossed as their score. If a 1 is tossed, all player's scores become 0, and the round is over. Players accumulate points each roll.
- Before each roll, each player can choose to end their turn. They acknowledge this by sitting down. In doing so, the points they have accumulated thus far become their score for that round. They are guaranteed to keep those points, but cannot continue to accumulate points on the rolls that follow. On the other hand, a player may choose to be a "Greedy Pig" and remain standing, risking their accumulated points for more possible points.
- Once a 1 is rolled, all remaining players that are standing lose their accumulated points for that round.

- Play for each round continues until a 1 is rolled, or all players have sat down.
- The objective is to have the highest grand total of points at the end of 5 rounds.

Lesson Outline

- Introduce the rules of the game. Discuss probability as it relates to the game.
  - What is the probability that you will score?
    $P(score) = \frac{5}{6}$
  - What is the probability that you will lose
    $P(lose) = \frac{1}{6}$
  - What is the probability that you will score on the first *and* second rolls? This can be found by writing out all possibilities for both rolls in a table.
    $P(\text{score on first and second rolls}) = \frac{5}{6} \cdot \frac{5}{6} = \left(\frac{5}{6}\right)^2$
- Play the Greedy Pig Game
- A discussion about independent and dependent events (if not already mentioned before in class) can be inserted here, in between playing the game twice. Other discussions can include strategies for playing the game. (Ex: Quit after a specific number of rolls, quit after a specific number of points, quit after a specific number of people have quit, never quit, etc.)
  - These rolls are *independent*. Future rolls **do not** depend on past rolls. Each and every time we roll, the probability of losing is always $\frac{1}{6}$. It could be the first roll, or the 100th roll; the probability is the same, because these events are independent. Many students will think that the chances of rolling a 1 increase with every roll, and this is an interesting point to discuss.

Extension: Expected Value

This section describes the expected value of an event, and relates it to an optimal strategy for playing the game.

Suppose you have 8 points accumulated. What could happen on the next roll?

- If a 1 is rolled you will lose 8 points. This will happen with probability $\frac{1}{6}$. If a 2 is rolled, you will gain 2 points. If a 3 is rolled, you will gain 3 points, etc. Each of these also has probability $\frac{1}{6}$.
- We can combine these possibilities into a weighted average, or *expected value*. We take each possible outcome, and multiply it by the probability that it will happen. Then, we add up all the possibilities.

$$-8\left(\frac{1}{6}\right) + 2\left(\frac{1}{6}\right) + 3\left(\frac{1}{6}\right) + 4\left(\frac{1}{6}\right) + 5\left(\frac{1}{6}\right) + 6\left(\frac{1}{6}\right) = \frac{-8+20}{6} = 2$$

Thus, you can expect to gain 2 points on the next roll.

Suppose you have 32 points accumulated. What could happen on the next roll?

- If a 1 is rolled you will lose 32 points. This will happen with probability $\frac{1}{6}$. If a 2 is rolled, you will gain 2 points. If a 3 is rolled, you will gain 3 points, etc. Each of these also has probability $\frac{1}{6}$.
- Again, we can combine these possibilities into a expected value:

$$-32\left(\frac{1}{6}\right)+2\left(\frac{1}{6}\right)+3\left(\frac{1}{6}\right)+4\left(\frac{1}{6}\right)+5\left(\frac{1}{6}\right)+6\left(\frac{1}{6}\right)=\frac{-32+20}{6}=-2$$

So you can expect to *lose* 2 points on the next roll.

Generalizing, if you currently have k points, then following the same calculation, your expected value for the next roll is

$$-k\left(\frac{1}{6}\right)+2\left(\frac{1}{6}\right)+3\left(\frac{1}{6}\right)+4\left(\frac{1}{6}\right)+5\left(\frac{1}{6}\right)+6\left(\frac{1}{6}\right)=\frac{-k+20}{6}$$

Notice that if $k > 20$, the expected value is positive, so we can expect to gain points, and if $k < 20$, our expected value is negative, so we can expect to lose points. Thus, one particular strategy for playing the Greedy Pig game could be to play until you reach 20 points, and then quit when your expectation becomes negative. A discussion here could include evaluation of this strategy, and development of other strategies.

Extensions could include assigning different point values to the numbers on the die (i.e. 2,3 and 4 are worth 2 points and 5 or 6 are worth 3 points), or playing with multiple die and using the sum of the die as the point value. These adjustments will change the probabilities and the expected value.