Ho‘olele Lupe

Introduction

The Hawaiian demigod Maui is given credit for the invention of the ho‘olele lupe (kite). Oral traditions, as well as the creation chants of Kaelikuahulu, suggest that ho‘olele lupe, made of hau covered with kapa or pandanus leaf, were an important part of Native Hawaiian society. Not only were they flown for recreational purposes by young and old men, but they were used for maritime propulsion, fishing, and fighting as well. Ho‘olele lupe might also have held religious significance, as evidenced by the effort the missionaries used to suppress its use (Ho‘olele Lupe-An Analysis of the Ancient Practice of Hawaiian Kite-flying, Damion Sailors).

Grade Levels and Topics

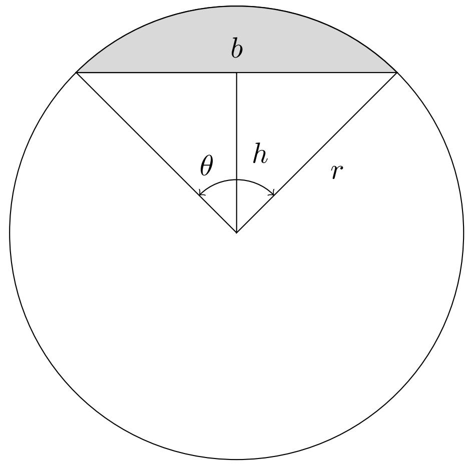
- **Geometry:**
  - **Finding areas of kites, circles, crescents/lunes, and hexagons:** The area of a kite and hexagon can be derived by using the formula for the area of a circle. The area of a circle is commonly known. The area formula for a crescent can be a bit complex, however there are some cases where the area of a crescent can be simplified; for example, look at number 7.2.1.d.i., it only requires you know the area formula for a circle.
    - ***Note***: the area of a lune/crescent A is:

$$A = 2\Delta + a^2 \sec^{-1}\left(\frac{2ac}{b^2 - a^2 - c^2}\right) - b^2 \sec^{-1}\left(\frac{2bc}{b^2 + c^2 - a^2}\right)$$

      Where a is the radius of the smaller circle, b is the radius of the bigger circle, c is the distance between the center of both circles, and

$$\Delta = \frac{1}{4}\sqrt{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}$$

      .
  - **Applying geometric formulas and theorems:** This lesson focus’ on formulas based on circles, triangles, cresents/lunes, ellipses, and hexagons. These formulas will be used to answer questions applicable to the real world.
  - **Triangles:** Pythagorean Theorem, similar triangles, special triangles ($30-60-90$ degree and $45-45-90$ degree), and areas.
  - **Area of a segment of a circle:** The formula for the following segment (pink) of a circle is:



$$A_{Segment} = A_{sector} - A_{triangle} = \left(\frac{\theta}{360}\right)\pi r^2 - \left(\frac{1}{2}\right)bh$$

Objectives and Outline

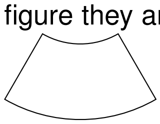
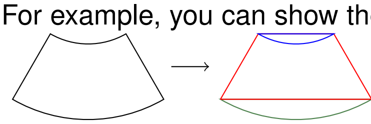
The overall purpose of this activity is to have students derive formulas and calculate areas for irregular shapes by decomposing the shapes into more familiar shapes. With the composite kite worksheet, students learn how to break down an irregular shape into simple shapes for which the area formulas are known. The students will then use this information in the future sections when they must find areas of irregular shapes. Similarly in the broken sections worksheet, students calculate the area of a simple shape with a missing portion. This offers another method of calculating the area of an irregular shape.

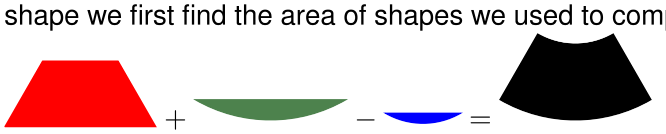
Contained in this PDF are worksheets for the students to practice calculating areas of unfamiliar shapes. To motivate the mathematics involved, students should be tasked with creating functions to describe the areas of the various kites in the worksheets. Then, with limited materials, the students can use their area function to determine how to build the kite. For example, the area of a kite (the geometric shape) is $\frac{d_1 \cdot d_2}{2}$, where d_1 and d_2 are the main diagonals. Armed with the knowledge of the total area of the kite materials, one can determine the length of d_1 and d_2 to maximize the area of the kite!

Materials

- For a list of materials to build a kite see this website: my-best-kite.com.
- Composite Shapes Worksheet.
- Broken Regions Worksheet.

Composing Kites Discussion

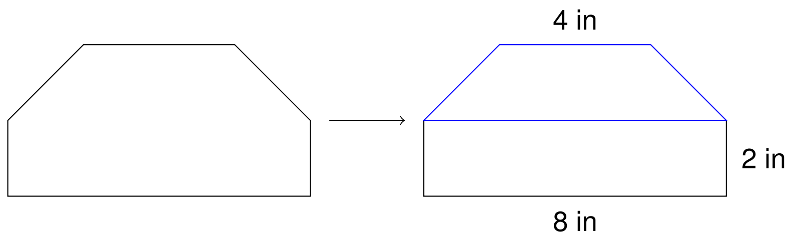
- Before delving into the activity, discuss with the students various shapes and their corresponding area formulas. They do not have to make a complete list, but have them compile a list of shape and area formulas.
- Discuss with the students what a composite shape is (**Note:** A *composite shape* is an object composed of two or more basic shapes). Ask the students if they can think of any composite shapes. What basic shapes make up that composite shape (e.g. t-shirt, human silhouette, side view of a building, etc...)?
- With good idea of composite shapes, have the students organize into groups and work on the composite shapes worksheet. Before the students start, inform them that their answers may not all be the same due to the fact that their are no numbers or units labeling the shapes. The purpose is to have the students create their own numbers (or better yet variables) and justify the areas they calculate. Encourage older students to use variables, as this will become more useful later. Facilitate this discussion: Which shapes were most difficult? Can we generalize a method? Are their multiple methods? Etc.
- To find the area of #6, students may need to take away overlapping areas in order to find the area of the figure they are working with. For example, you can show the students how to find the area of this shape  we can change  . And then to find the area of the original shape we first find the area of shapes we used to comprise the image:



- Instead of students having to remember complex area formulas, this method allows them to formulate their own area formulas in the future.

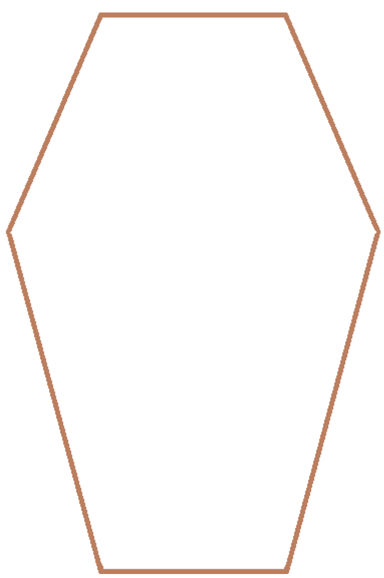
Composite Shapes

Change the following ho‘olele lupe shapes into composite shapes. Then create an approximate area formula using numbers (or variables for a challenge). Example:



$$Area = 8 \cdot 2 + \frac{8 \cdot 4}{2} = 32\text{in}^2$$

1.



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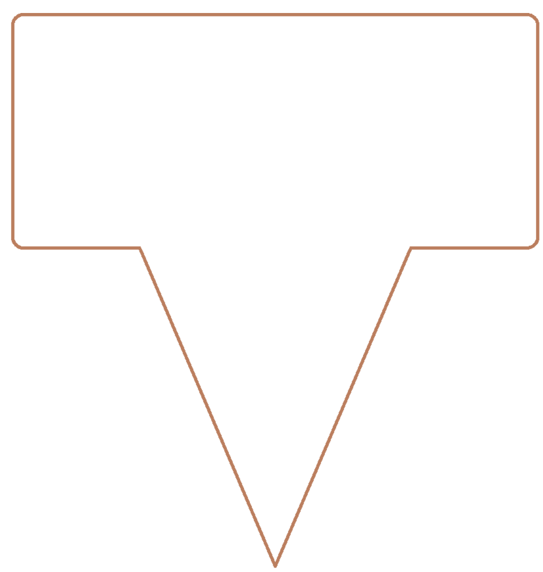

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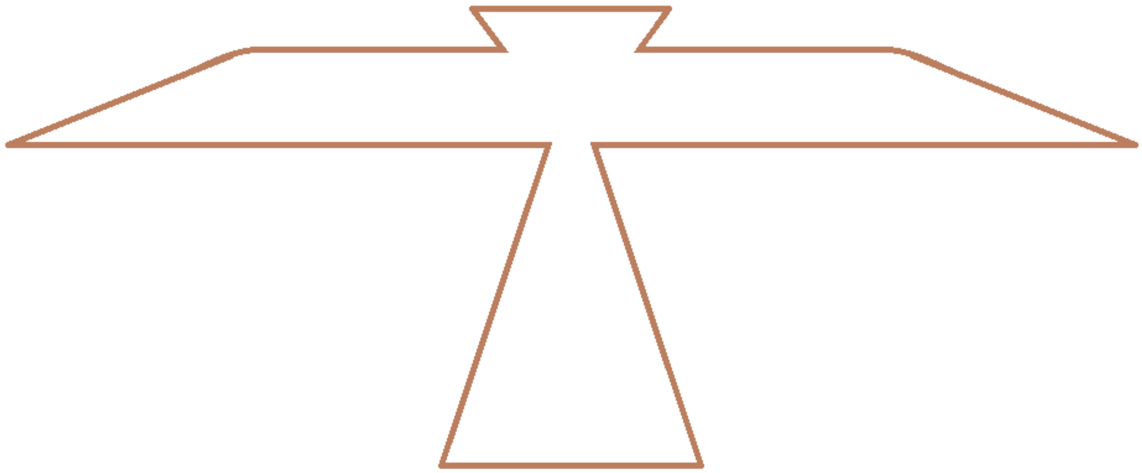

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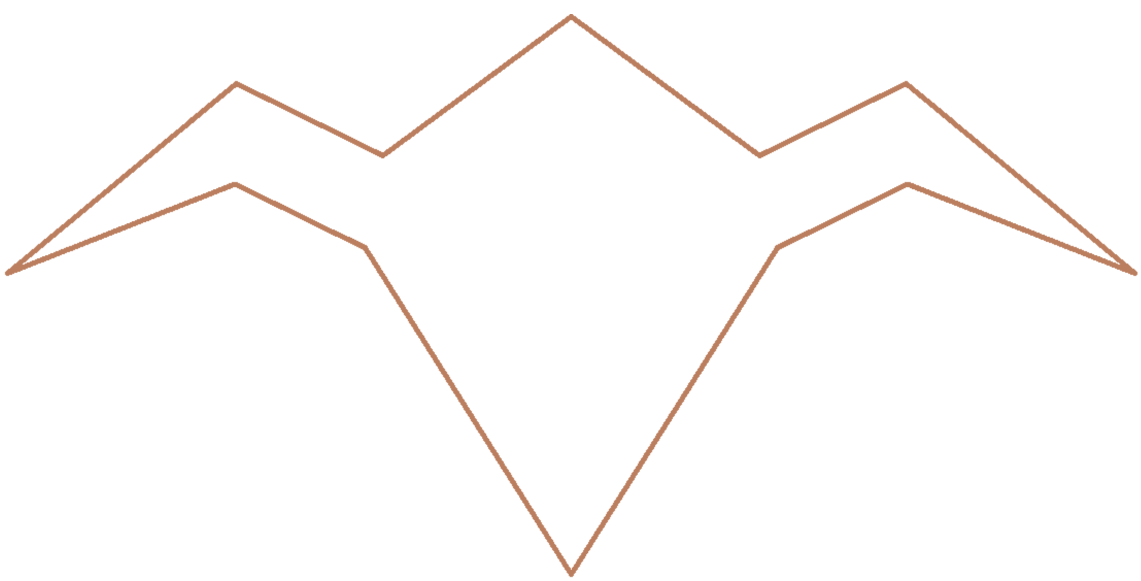

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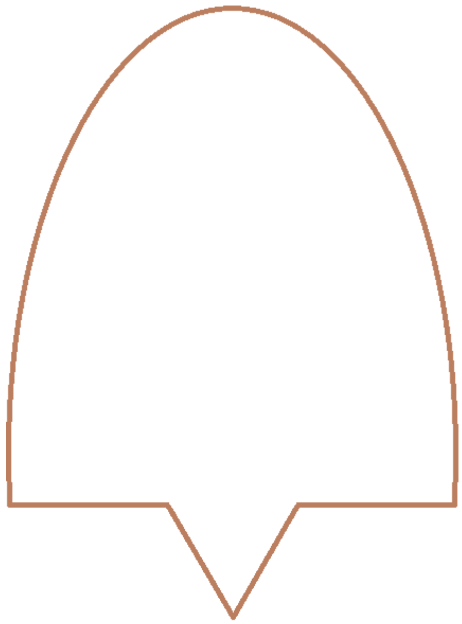

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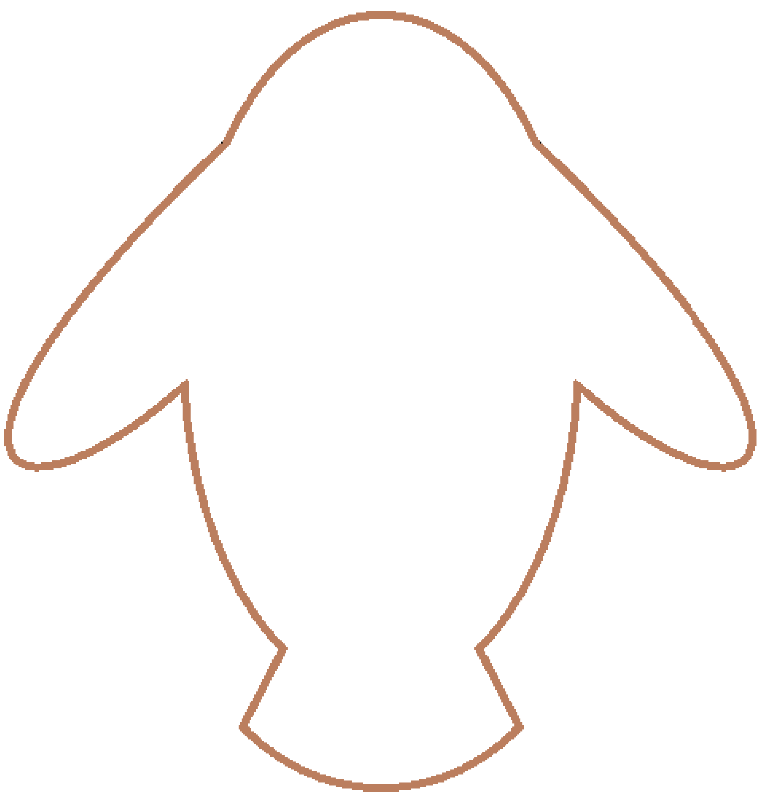


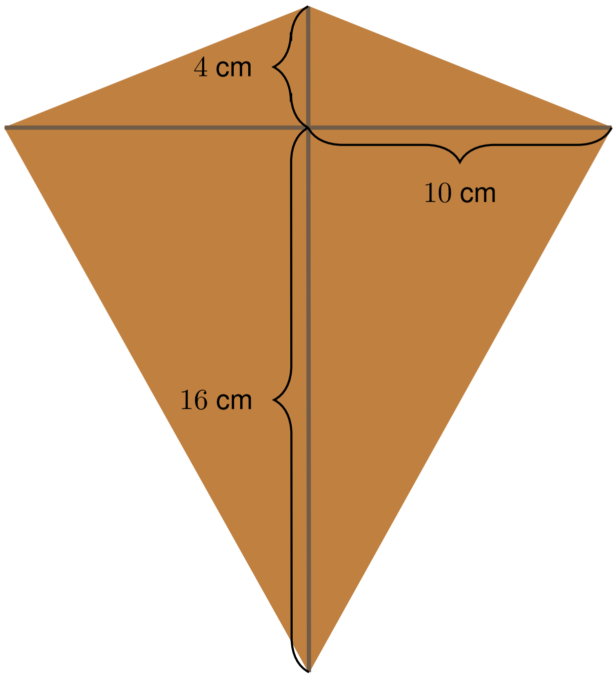
Broken Regions Discussion

- Discuss with the students how to derive a formula for the area of a kite and hexagon.
- With the students in groups, have them derive their own formulas for the areas of a circle segment, sector of a circle, and the broken regions of the ho‘olele lupe in the worksheet below.
- For problem 1 part d-iii, discuss with them how you can use can use the Cartesian plane to organize geometric data.

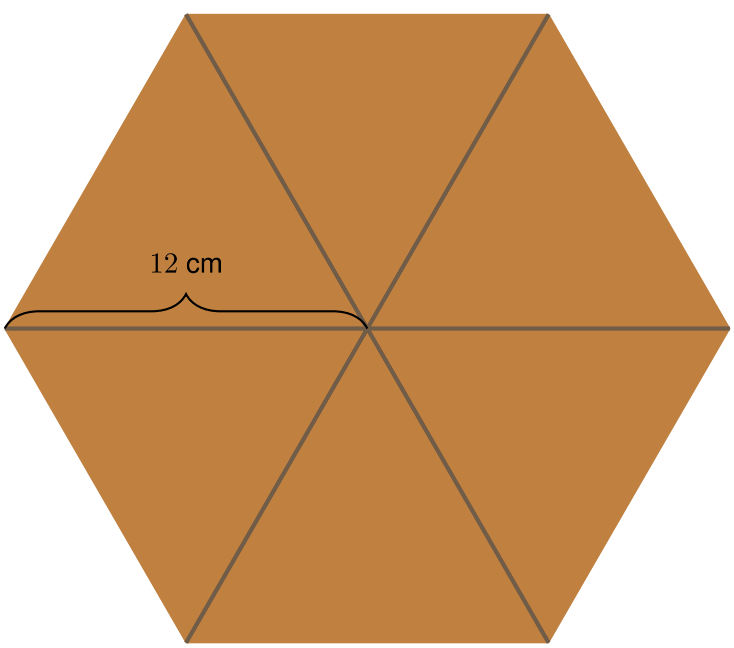
Broken Regions

1) Determine the area of the following ho‘olele lupe.

(a) Kite shaped ho‘olele lupe:

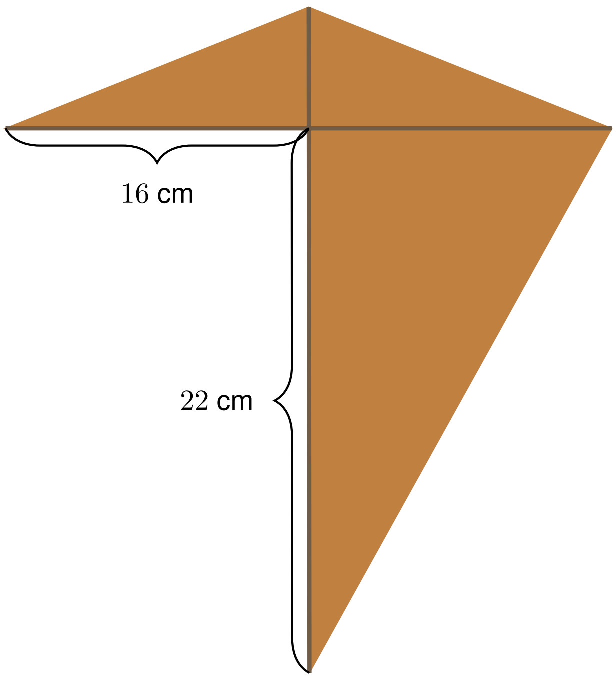


(b) Hexagonal ho‘olele lupe:

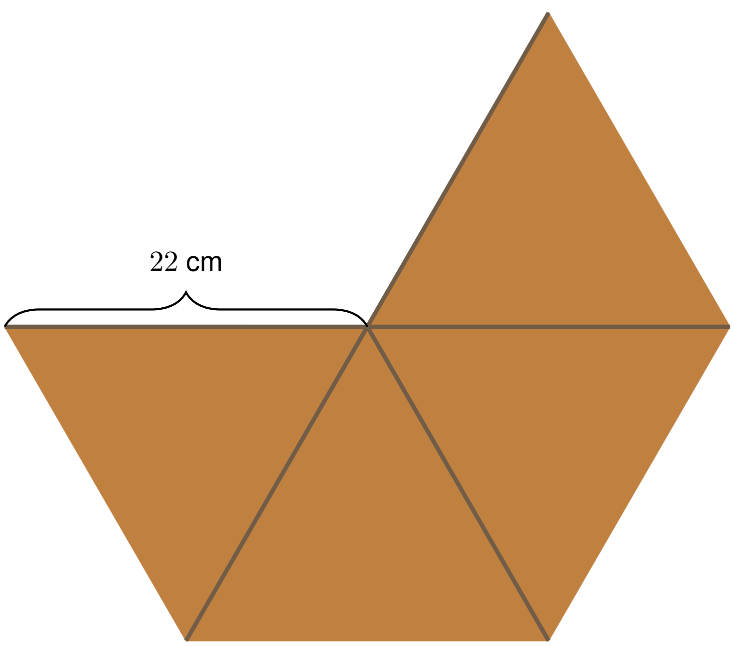


2) For each broken hoʻolele lupe, determine the amount of *kapa* required to repair the missing piece.

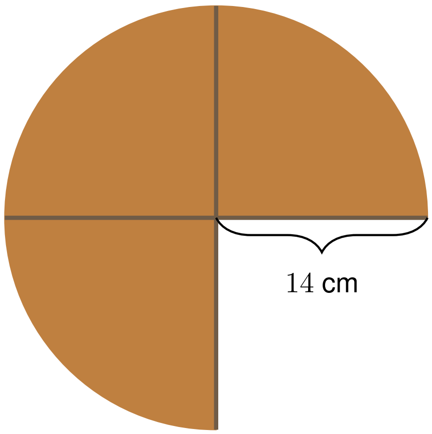
(a) Kite shaped hoʻolele lupe:



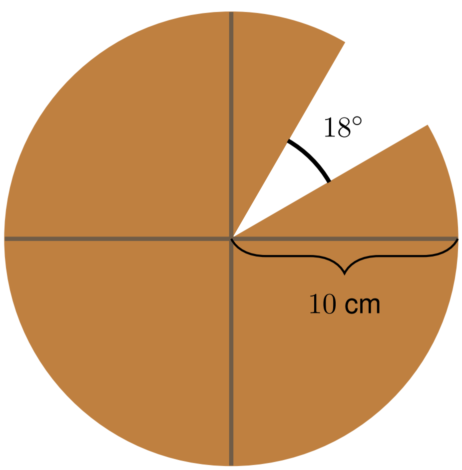
(b) Hexagonal hoʻolele lupe:



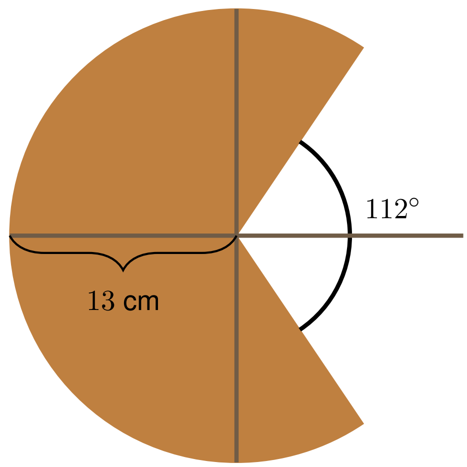
(c) Circular hoʻolele lupe:



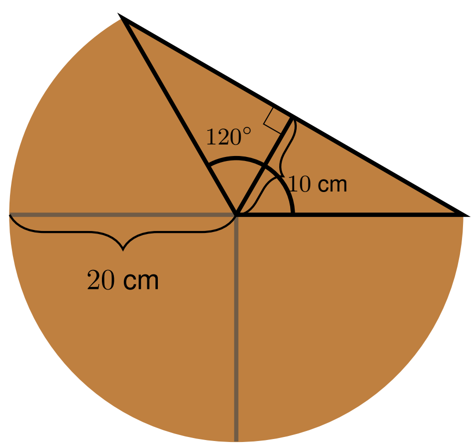
(d) Circular hoʻolele lupe:



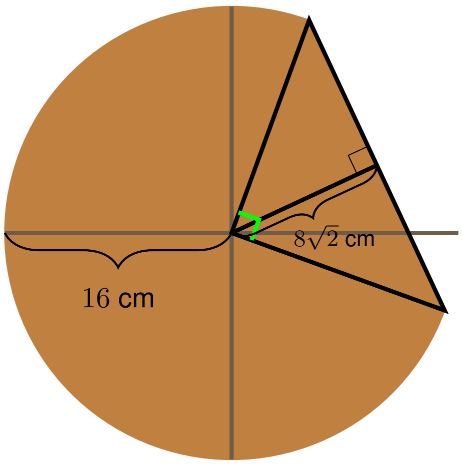
(e) Circular hoʻolele lupe:



(f) Circular hoʻolele lupe:

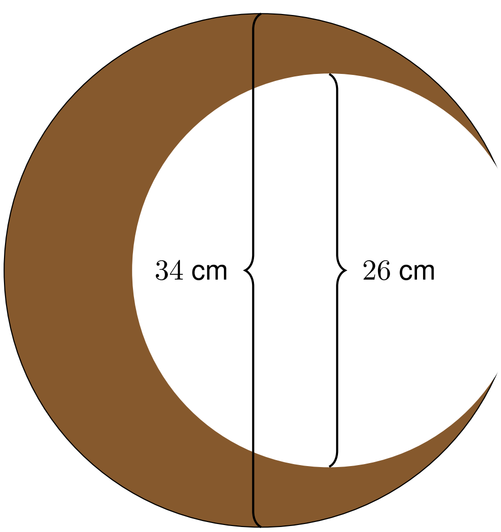


(g) Circular ho‘olele lupe:

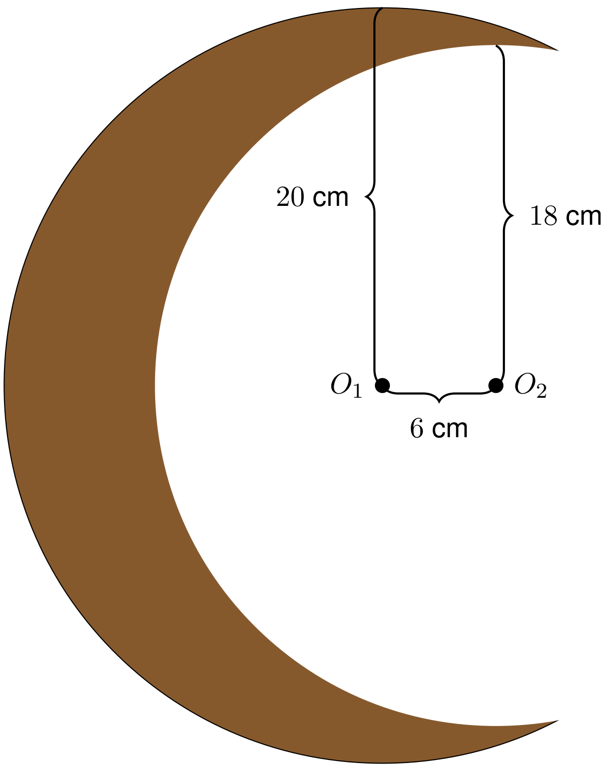


3) Crescent ho‘olele lupe:

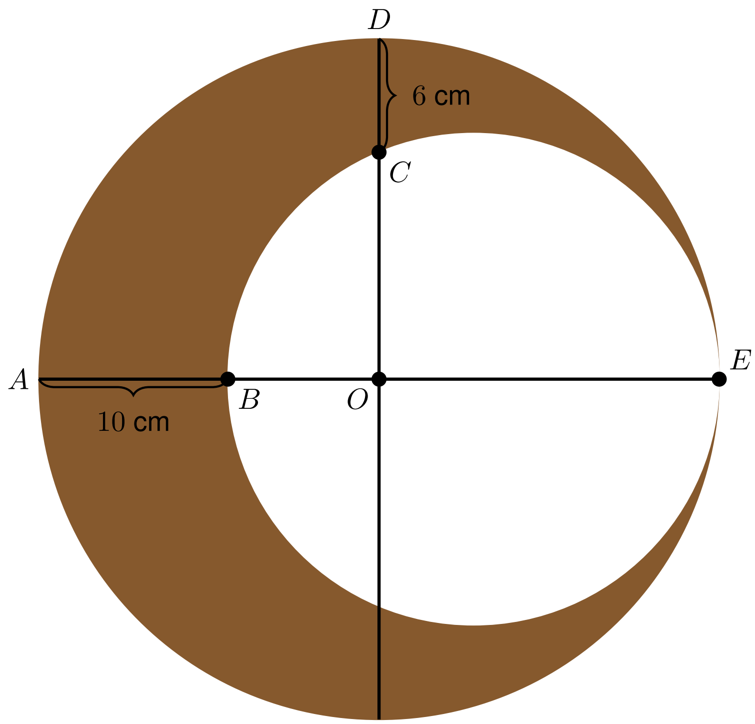
(a)



(b) O_1 and O_2 are the centers of the bigger and smaller circle respectively.



(c) $\overline{AE}$ and $\overline{BE}$ are the diameters of the bigger circle and smaller circle respectively.[Hint: use the fact that triangle BOC and BCE are similar triangles].



4) Create a diagram of a ho‘olele lupe you would like to make. With your diagram, determine the amount of materials needed to complete the kite.

Answer Key

Broken Regions Answers

1. (a) 200 cm^2
   (b) $216\sqrt{3}$ cm^2
2. (a) 176cm^2
   (b) $242\sqrt{3}$ cm^2
   (c) 49πcm^2 ≈ 153.94cm^2
   (d) 5πcm^2 ≈ 15.71cm^2
   (e) $(\frac{2366}{45})\pi$cm^2 ≈ 165.178cm^2
   (f) $\left(\frac{1}{3}\right)(20^2\pi) - 100\sqrt{3} \approx 245.67$cm^2
   (g) $\left(\frac{1}{4}\right)(16^2\pi) - 128 \approx 73.06$cm^2
3. (a) 120.61 cm^2
   (b) 486.95 cm^2
   (c) $18^2\pi - 13^2\pi = 155\pi \approx 486.95$cm^2