Probability Bag

Grade Levels

This lesson is designed for grades 6-8. It takes one class period to run.

Objectives

The objective of this lesson is to gain experience in data sampling. With the samples, the students will use ratios and probability to infer on the makeup of a population.

Materials

Counters, two colors *(quantity discussed in next section)*
Small bags *(quantity discussed in next section)*

Preparation

In this lesson plan, we will be using black and red chips as our counters, but any type of counter will work as long as there are two types. The students will be sampling from a bag of chips, returning the chip sample to the bag before sampling some more. All students will pool their sampling data in the end. Given the sampling data and the total number of chips in the bag, the students will infer the approximate number of black or red chips in the bag. To increase the rate of sampling and to ensure randomness between samples, multiple identical bags will be passed around the classroom during sampling. To prepare these bags, use the following table.

Item	**Instructions/notes**	
Number of students		A
Number of bags	Recommended: one bag for every four students	B
Counters per bag	Recommended: about 30 counters	C
Black counters		D
Red counters	$D + E = C$	E
Rounds of sampling	Choose F such that $A \times F > 230$	F

Instructions: Prepare B paper bags with D black and E red counters. *For bonus theatrics,* calculate $\frac{A\times F\times D}{C}$, write this number on a sheet of paper and seal this in an envelope. This is the predicted number of black counters in the class sample. Note that the nature of probability activities suggest that unlikely events can happen. So please be aware that the final conclusion of this activity may not be as expected. We improve the likelihood of an expected conclusion with our above recommendations (e.g. choosing F such that $A\times F > 230$).

These recommendations come from computer simulations of this activity. The occurrence of unlikely events are a fundamental aspect of probability. So please don't shy away from discussing it if/when it comes up.

Activity

We don't always notice it, but sampling is an integral part of our local culture. When we share food with our friends and family, we sometimes say in pidgin "eh, like try sample?" Let's consider what our intentions are when we say this. When we offer a "sample" to our friends while holding a sandwich, is it okay for our friends to eat the whole sandwich? Usually, no–we expect our friends to only take a small piece. So why do we share food in quantities that are never enough satisfy someone's hunger? Usually it is because we want others to experience the food and to gain some insight on what it is like to have the whole meal. In other words, we want our friends to taste a piece of our sandwich so that they can guess what the whole sandwich is like. This is exactly what sampling is–we take a sample from a population and from the sample, we make inferences on the population. The inferences we make depend on the samples we obtain. For example, if by chance, we only taste bread in our sample, then we could guess that sandwich is mostly, if not completely made of bread. If we taste pork instead, then we may infer that the sandwich is mostly, if not completely made of pork. As you may guess, one way to improve our guess is by taking more samples. For example, suppose we take five bites (samples) from the sandwich (population). If one of the bites is bread and the rest of the bites are pork, then we may infer that the sandwich is probably made up of 20% bread and 80% pork. It must be noted that it is possible to take five bites and miss the pork every time, thus leading you to make the wrong inference. However, this erroneous inference is still much less likely than if you only took one bite. Statistics and probability are fields of mathematics that help determine how trustworthy a guess is and how a guess can be improved.

After explaining the purpose of sampling, have the students create the following data sheet.

My data
Black counters _______________ *Red counters* _______________
Total counters _______________

Now distribute the B number of bags evenly throughout the classroom. The students who are holding the bags should repeat the following five steps:
1. Grab one counter from the bag.
2. Record its color.
3. Return the counter to the bag.
4. Shake the bag.
5. Pass the bag to the next student.

Continue this until the total counters that *each* student has recorded is equal to F. Now post all of the student data in the front of the classroom. Have the students calculate the total number of counters (which should be equal to $A \times F$), the total black and the total red counters. Then have each student fill out the following data sheet.

Class data
Black counters _______________ *Red counters* _______________
Total counters _______________

If you put a number into a sealed envelope beforehand, now is a good time to open it and brag about how close it is to the total number of black counters in the class sample.

Now, inform the students that each bag has a counter total of C. Challenge the students to figure out how many of those counters are black. If needed, ask the following questions as hints.
What fraction of the sample is black?
What fraction of counters in the bag is black?

When the students have come to a consensus on how many black counters are in the bag, conclude the activity by allowing them to open the bag and examine its contents.