# Let’s Make a Deal

## Grade Levels

This activity is intended for grades $6-8$.

## Objectives and Topics

Through this lesson, students learn and construct a solution to the famous “Monty Hall Problem”. This problem illustrates precisely how in the event of randomness, our intuition fails us, but not the mathematics! Students will gain practice applying the multiplication rule of probability as well as more familiarity with independent/dependent events.

## Scenario

You are on a game show and you’re given the choice of three doors. Behind one door is a brand new luxury car; behind the others, goats. The car and the goats were placed randomly behind the doors before the show. The rules of the game show are as follows: After you have chosen a door, the door remains closed for the time being. The game show host, who knows what is behind the doors, now has to open one of the two remaining doors, and the door he opens must have a goat behind it. If both remaining doors have goats behind them, he chooses one randomly. After the host opens a door with a goat, he will ask you to decide whether you want to stay with your first choice or to switch to the last remaining door. Image that you chose Door $1$ and the host opens Door $3$, which has a goat. He then asks you "Do you want to switch to Door Number $2$?"

After just reading the problem, do you think one option is better than the other? If so, which choice is to your advantage?

## Simulation

We will now simulate this game to try to determine the answer. Each pair will need three Dixie cups, and a starburst. **DO NOT EAT THE CANDY JUST YET!** You and your partner will take turns as the host and the player.

The host will switch around the three cups to randomize them. Then, the host will look under the cups so they know which cup has the starburst under it. The player then chooses one cup. The host then uncovers a cup with nothing underneath. If the player chose the cup with the candy, the host should randomly pick a cup to uncover.

Each person should play the game ten times. Switch from your choice five times and stay with your original choice five times. Record your results:

| | | |
|---|---|---|
| Game 1 | Strategy: Switch | Outcome: |
| Game 2 | Strategy: Switch | Outcome: |
| Game 3 | Strategy: Switch | Outcome: |
| Game 4 | Strategy: Switch | Outcome: |
| Game 5 | Strategy: Switch | Outcome: |
| Game 6 | Strategy: Original | Outcome: |
| Game 7 | Strategy: Original | Outcome: |
| Game 8 | Strategy: Original | Outcome: |
| Game 9 | Strategy: Original | Outcome: |
| Game 10 | Strategy: Original | Outcome: |

Games won when switching: Games won when staying:

## The Monty Hall Problem

### Background

The question given above is a mathematically rigorous version of one published in *Parade* magazine in $1990$. The problem is named after the host of the old game show *Let's Make a Deal*, because the puzzle is similar (but not identical to) the game played on the show. This problem, and other puzzles that are identical mathematically but might use a different story, are famous because it is rare to find a person who gives the correct solution when first confronted by them (the author was stumped the first time he tried it).

Read through the problem again and try to figure out the probability of winning if you switch and the probability of winning if you stay before reading on.

If you answered that the probability of winning is $\frac{1}{2}$, no matter which strategy you choose, you are in good company. The author and the numerous PH.D.'s in Mathematics and experimental sciences have all given the same response. That does not make it any less false.

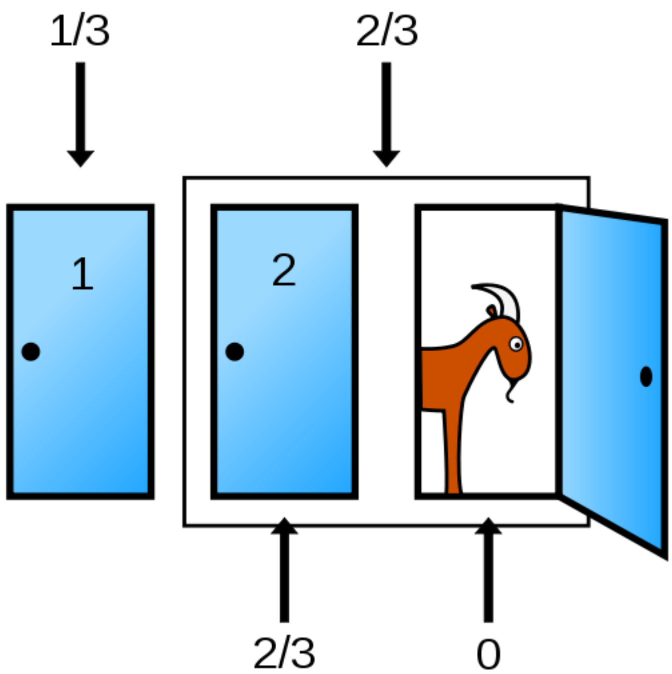
At first the problem seems obvious: after making your initial choice and having the host open a door, there are two doors remaining and one of them has the prize behind it, so the odds of the door you pick having the prize are $\frac{1}{2}$. But this doesn't take into account the way that the host chose which door to open. There are several ways to look at the problem to see the correct solution.

### Method 1

Knowing that the prize was randomly placed before the game, the odds that the door you initially pick has the prize behind it is $\frac{1}{3}$. If your initial choice was correct, then you will win the game by staying with your original choice. Regardless, of your original choice, the host must open a remaining door with a goat behind it. If you chose wrong initially, you will win if you switch. Thus, by staying with your original choice you will win only those games when you successfully guess which door the car is behind with your original choice, $\frac{1}{3}$ of the time. On the other hand, by switching you will win those games when your first choice did not have the car behind it, the other $\frac{2}{3}$ of the time.

### Method 2

Because the host has to open a door with a goat behind it, he is essentially giving you the choice between $1$ door (your initial choice), or $2$ doors (if you switch). Because the car and goats were distributed randomly before the game begins, there is a $\frac{1}{3}$ chance of the prize being behind your initial choice, and thus a $\frac{2}{3}$ chance of the prize behind behind one of the two remaining doors. After the host opens a door other than your initial choice to reveal a goat, there is still a $\frac{2}{3}$ chance that the prize is behind one of the two doors that you did not choose initially, but obviously there is $0$ chance it is behind the door the host opened. Thus, there is a $\frac{2}{3}$ chance the prize is behind the door that you did NOT choose.




### Method 3

Using the picture above, one can make a chart showing the possible decisions at each point in the game and probability that each decision is made. The total probability of a given outcome can then be found by taking the product of the probabilities of each decision at every point on the chart leading to the outcome. This chart illustrates the process if the car is behind door $1$.

| Player's Choice | Host Opens | Total Probability | Stay | Switch |
| --- | --- | --- | --- | --- |
| Door 1 | Door 2 | $\frac{1}{6}$ | Car | Goat |
| Door 1 | Door 3 | $\frac{1}{6}$ | Car | Goat |
| Door 2 | Door 3 | $\frac{1}{3}$ | Goat | Car |
| Door 3 | Door 2 | $\frac{1}{3}$ | Goat | Car |

Make the charts for the game's probabilities when the car is behind doors $2$ and $3$ and you will see a similar pattern. In each case, $\frac{2}{3}$ of the time one wins by switching, while one wins only $\frac{1}{3}$ of the time by staying.

## Teaching Monty Hall

The game can be easily simulated with something else under the cups, or cards of some kind. Give the students the handout and have them read the problem (out loud often helps). Answer any questions if students are confused about how the game operates. Demonstrate the game as the host with one or more of the students a few times to be sure everyone knows what to do. Then, have the class, either alone or in groups, consider whether there is a winning strategy.

Have the students pair off (you may have to participate as well to make up the numbers) and distribute the manipulatives. Instruct the students that each person should be the contestant $10$ times and be the host $10$ times. Each student should record the results of the games where they are the *contestant*, being sure to switch $5$ times and stay $5$ times.

When all the students have finished simulating the game, pool the class data on wins and losses for each strategy. You can do this on the board or on a spreadsheet program if you have a projector. As long as there are a fair number of students who participated ($15-20$ is plenty) the pattern of winning significantly more games when switching should be apparent.

Have the students discuss why switching might be a better strategy in small groups and then report their conclusions to the class. The groups will probably come up with explanations that are similar to those given above. Try to lead the class discussion to complete these explanations for the problem. It is best if you can lead class discussions about each viewpoint of the problem presented above as well as any others the students may have thought of. This is a very non-intuitive puzzle and it always helps to present many ways of thinking about it. Conclude by summarizing that despite initial appearances, the simulation carried out by the students revealed that switching was by far the best strategy for the game and that there are logical explanations for why this is true.