The Rosette

Grade Levels

This activate is intended for students in grades 7–12.

Objectives and Topics

This activity shows students how the perimeter of a disc is proportional to its radius and the area depends on the function of the perimeter and its radius. These approach the value of pi. This activity is best used if students have not already seen the formulas for the perimeter and area of a disc, but it is still nice to show students where these formulas originate from.

Materials and Resources

- Rectangle cutouts (see below)

Introduction: The Value of π

Since the days of antiquity, mathematicians have sought to calculate the perimeter of a circle - or the area of a circle - depending on its diameter. Even the earliest calculations used a constant ratio:

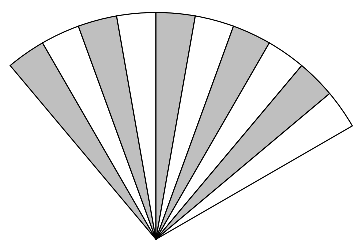
- Babylon (circa 2000 BC), this ratio had a value of $3 + 7/60 + 30/3600$ or 3.125
- In Egypt (1650 BC), a geometric method used amounted to the value as being equal to about 3.16
- In the Old Testament, data suggests the used value was 3.
- In Greece (c. 250 BC), Archimedes calculates the length of the circle enclosed by two regular polygons, one inscribed and one circumscribed. Using polygons with 96 sides, he obtained a ratio of between $223/71$ and $22/7$, or about 3.1408 and 3.1428. The value of $22/7$ will long remain as an accepted, satisfactory approximation of pi (Note that this value is what appears when doing a rosette with rectangles 7 inches wide!).

Subsequently, more and more precise values are obtained in China, India, the Middle East, and Europe during the Middle Ages. The development of techniques for calculation, from the 17th century, eventually made the approach to π more precise: 16 digits in 1665 (Newton), 100 decimal places in 1706 (Machin). Computers have recently allowed mathematicians to pulverize these records: 4 billion digits in 1994, and over 1 trillion today. In this activity, students will find their own approximation for pi and use it to create the formulas for the area and parameter of a circle.

Outline

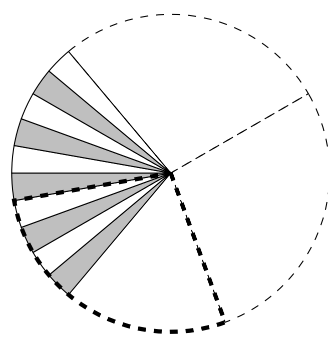
Working in groups of two, each group gets a rectangle and a sheet statement. Distribute the 6 different rectangles amongst the groups. Have the students cut out and fold the rectangles along the dashed lines. If necessary, show how to mark the folds with rulers and scissors to make them accurate. Discuss with the students how they believe the rectangles can be used to approximate the area/perimeter of a circle. Allow the students to formulate questions and debate their validity while pooling the information.

After this discussion, have the students pinch one end of the "accordion" and open the the opposite end to create a "fan" (see picture below).



Pinch one end of the accordion, then open it to form the largest possible range

Now, their next task is to calculate the length (note: the width is the radius of the circle to be formed) their rectangles need to be in order to close the rosette! See the notes section below for possible approaches.



What should be the length of your rectangle so that you can make the rosette closed?

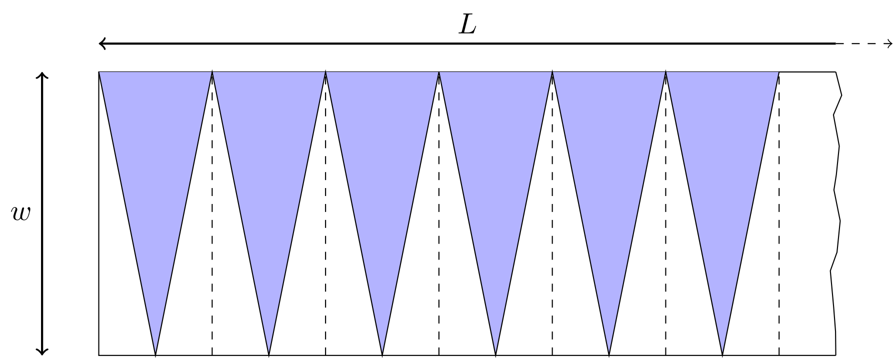
Have them take measurements of their rectangles and calculate the lengths their rectangles need to be in order to make a full disk out of the "fans". Once every group has an idea of the required length, challenge them to discover a common pattern between each other group's rosette. The students should have a constant ratio between width and length of a rectangle and its length compared theoretically to 2π, but will not be accurate. They should be approximately 31cm for the 5cm, 44cm for the 7cm and 63cm for the 10cm.

The significant differences in lengths show that they are based on the width (the radius of the circle) of the rectangles. If there are significant differences between the results for the same width, have the students clarify their procedures to determine which are the most reliable, then the various groups measure again to refine them. Suggest to fix one end of the "accordion" with tape to improve accuracy. If necessary, all groups can begin the search with the same width to obtain a more accurate result.

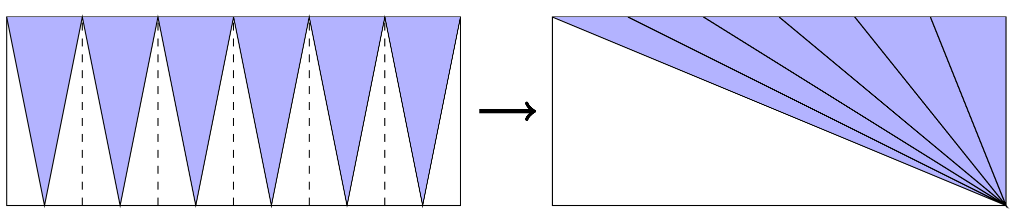
At the end of this collective research, present π and its value, equal to half of the constant found since it expresses the ratio between the perimeter of the circle (the length of the "extended" rectangle) to its diameter (twice the width of the rectangle), mathematically: $2\pi \approx \frac{L}{w}$. Finding the area of the disc is the second activity, after agreeing on the relationship between the sides of the rectangle.

Notes

- Possible approaches to search the length of the rectangle:
  - Extend the rectangle to form a semi-disk and multiply by 2.
  - Draw the angle, then follow it until the disc is closed.
  - Calculate proportionality by measuring the angle.
  - Count the number of folds contained in a right angle, then multiply by 4.
- Find the area of the disc from the "extended" rectangle:
  - Calculate the area of a triangle ($\approx \frac{1}{2}$ cm) and multiply by the number of triangles in the "extended" rectangle.



  - Calculate the area of a rectangle and divide by 2 (below).



We can do this because the area of a triangle only depends on the height and width.
Note that we did not change the width or height