The Greedy Pig Game

Instructions

1. A game can consists of 5 rounds

2. Each round consists of a series of rolls. A die is tossed. If a 2-6 is tossed, players write down the corresponding number as their score. If a 1 is tossed, all player's scores become 0, and the round is over. Players accumulate points each roll.

3. Before each roll, each player can choose to end their turn. In doing so, the points they have accumulated thus far becomes their score, and they will no longer continue accumulating points on the rolls that follow. On the other hand, a player may choose to risk their accumulated points and keep rolling the die to add to their score.

4. Once a 1 is rolled, all remaining players lose their accumulated points, and their score for the round is 0. Players who had previously ended their turn keep their accumulated points.

5. Play for each round continues until a 1 is rolled, or all players have voluntarily ended their turn.

6. The objective is to have the highest grand total of points at the end of 5 rounds.

Think About Your Strategy

1. Quit after n throws

2. Quit after you have accumulated n points

3. Quit after n people or n percent of people have quit

4. Play risky and never quit, hoping everyone else will quit

5. A different strategy??

The Math Involved

- What is the probability that you will score? $P(score) = \frac{5}{6}$ What is the probability that you will lose all your points? $P(1 \text{ is rolled}) = \frac{1}{6}$
- What is the probability that you will score on the first *and* second rolls? $P(2 \text{ scores in a row}) = (\frac{5}{6})^2$ What is the probability that you will score on the first n rolls? $P(n) = (\frac{5}{6})^n$
- These rolls are *independent*. If a 5 was rolled last turn, does that change the probability that a 5 will be rolled this turn or next turn? *No.*

One possible strategy: *Finding an Optimal Number of Points*
Say you have 8 points collected. What could happen on your next roll?

- If a 1 is rolled, you will lose 8 points. That will happen with probability 1/6. If a 2 is rolled, you will gain 2 points, if a 3 is rolled, you will gain 3 points, etc. Each of these also has probability 1/6.
- Combining those possibilities is what we call an *expectation*, which is sort of like a weighted average. Each outcome is multiplied by the probability of it happening, and then we add up all the possibilities: $-8(\frac{1}{6}) + 2(\frac{1}{6}) + 3(\frac{1}{6}) + 4(\frac{1}{6}) + 5(\frac{1}{6}) + 6(\frac{1}{6}) = \frac{-8+20}{6} = 2$. So you could expect to gain 2 points on the next roll.

Say you have 32 points collected. What could happen on your next roll?

- If a 1 is rolled, you will lose 32 points. That will happen with probability 1/6. If a 2 is rolled, you will gain 2 points, if a 3 is rolled, you will gain 3 points, etc. Each of these also has probability 1/6.
- The expectation this time is: $-32(\frac{1}{6}) + 2(\frac{1}{6}) + 3(\frac{1}{6}) + 4(\frac{1}{6}) + 5(\frac{1}{6}) + 6(\frac{1}{6}) = \frac{-32+20}{6} = -2$. So you could expect to lose 2 points on the next roll.

Let's generalize the situation. Instead of 8 or 32 points, you have k points. We can see that our expectation for the next roll is: $-k(\frac{1}{6}) + 2(\frac{1}{6}) + 3(\frac{1}{6}) + 4(\frac{1}{6}) + 5(\frac{1}{6}) + 6(\frac{1}{6}) = \frac{-k+20}{6}$. Notice that if $k > 20$, our expectation is to lose points, but if $k < 20$ our expectation is to gain points. Many people use this strategy to stay in the game until they have reached 20 points, and then quit.
Do you think this is a good strategy? Can you think of any situations where this strategy might not be the best choice?